Universitatea Politehnica Timişoara
Facultatea de Construcţii
Departamentul de Construcţii Metalice şi Mecanica Construcţiilor

COMPOSITE STEEL-CONCRETE STRUCTURES

- CURS 2 -

Composite Beams

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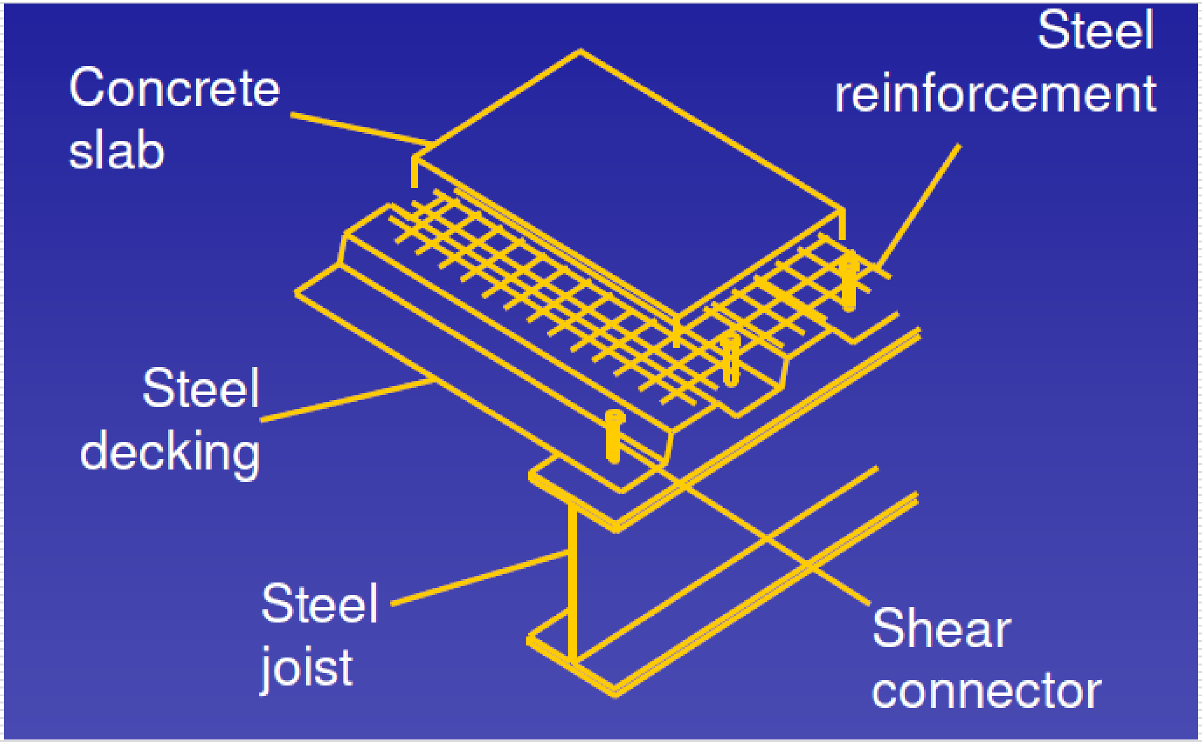
CHAPTER II – COMPOSITE BEAMS

§ 2.1 Introduction

- **Composite beams** are defined as "elements resisting only flexure and shear that comprise two longitudinal components connected together either continuously or by a series of discrete connectors".

- It is assumed that the two components are positioned directly one above each other with their respective centroids vertically above each other.



§ 2.1 Introduction

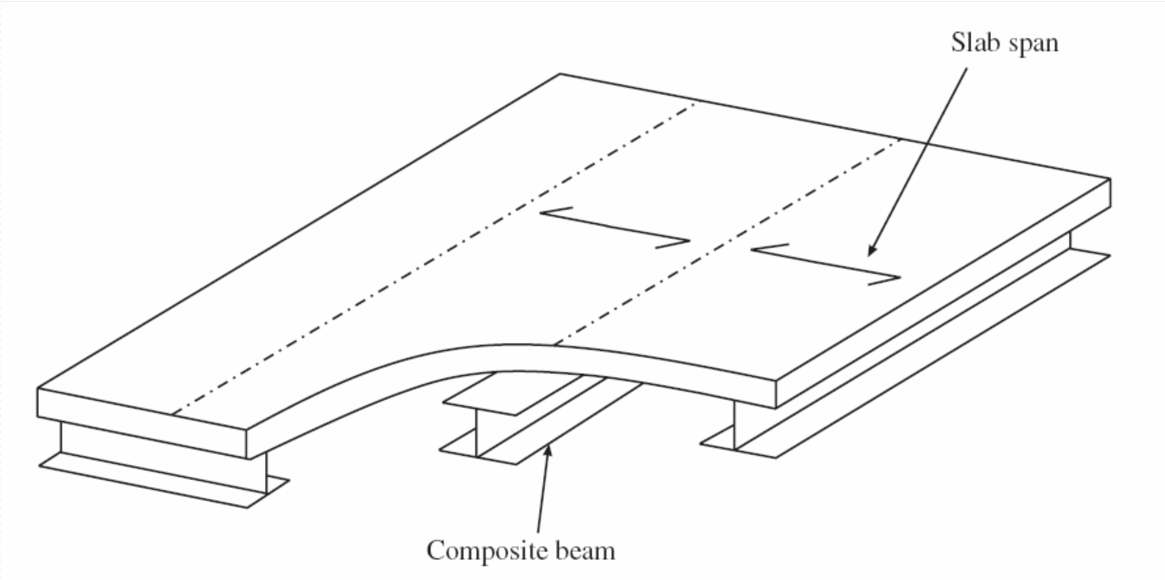
- Composite beams vary in behaviour from the situation when the connection between the two layers is non - existent to the situation where the bond between the layers approaches infinite stiffness and strength. There is also the influence of the contrast in material properties of the two layers.

- The influence of the difference in strength and stiffness of the components and the strength and stiffness of the connection between them plays a vital role.
- In consequence, the overall analysis and design of composite beams is, therefore, significantly more complex than for single material beams.

Obs: Critical to the overall behaviour of composite beams is the specific behaviour of the connection.

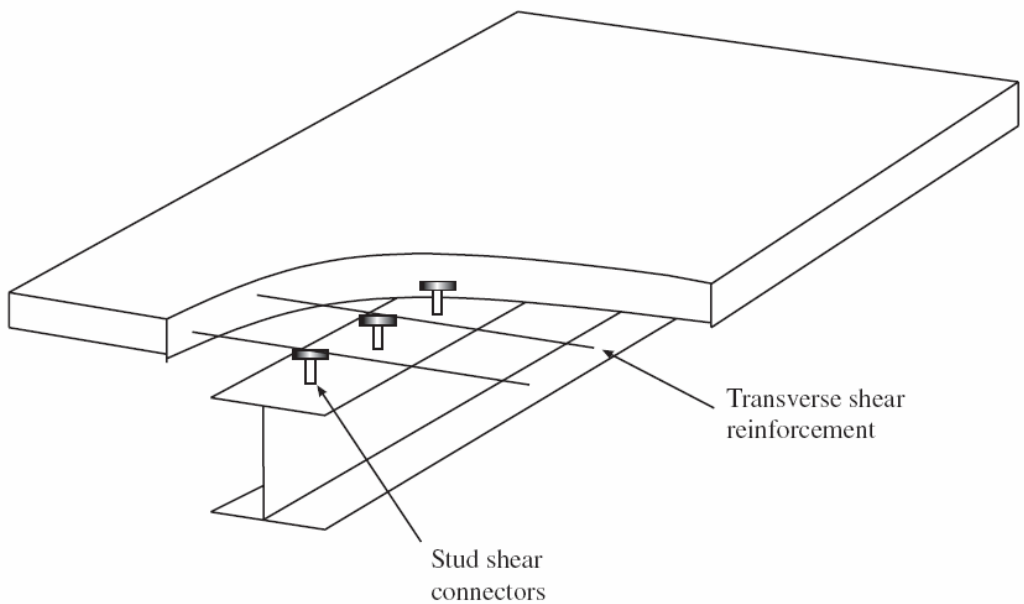
§ 2.2 Types of beams

- The generic form of composite beam comprises the combination of a solid concrete slab attached to a rolled steel section (normally of I shape). The slab will be designed to carry the floor load spanning between parallel beams but may also take compression perpendicular and along the beam line if it is connected to the steel section. This arrangement is shown in figure below.



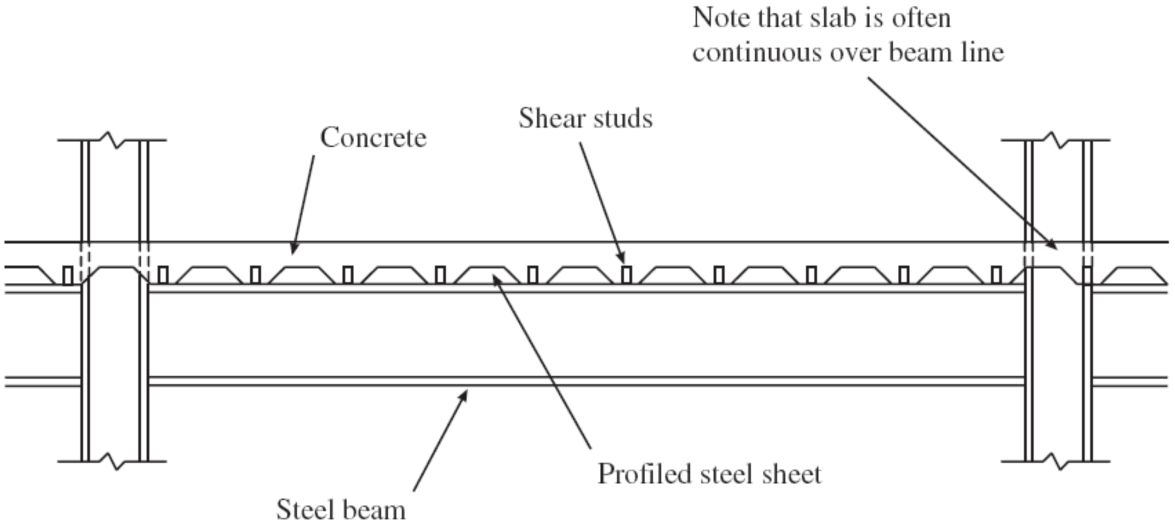
§ 2.2 Types of beams

- The connection between steel and concrete must be sufficient to control longitudinal shear and any uplift forces. The longitudinal forces generated by this connection must transfer fully from the steel section into the wider slab.
- Shear stud connectors and typical transverse shear reinforcement is shown in figure below.



§ 2.2 Types of beams

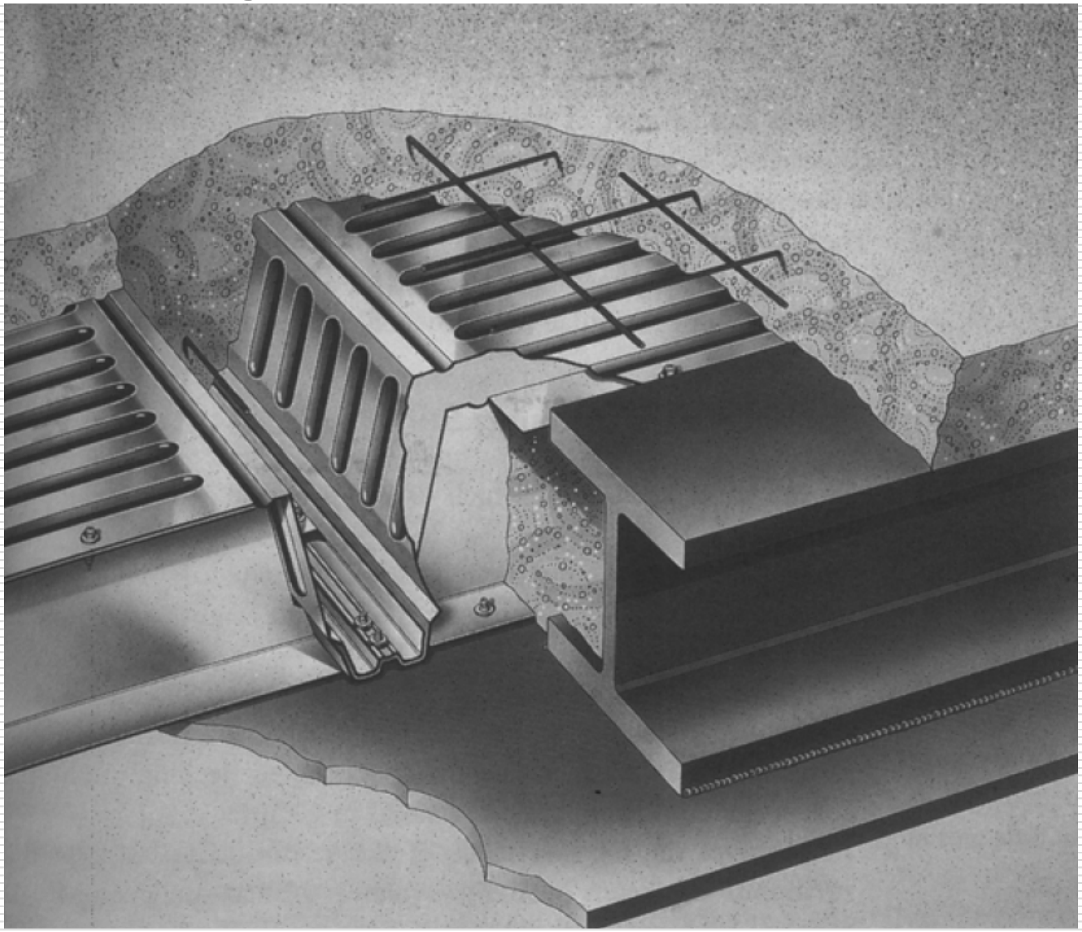
- The concrete slab may be a composite deck. In this case the connection is more difficult. The predominant form of connection is the “Through Deck” welded stud. This construction uses high output arc and forge weld equipment to burn through the steel sheet and weld the stud to the beam below. Figure below shows a typical composite beam formed with a composite slab.



Obs: The ribs of the profiled sheeting could be oriented either parallel or perpendicular to the steel beam.

§ 2.2 Types of beams

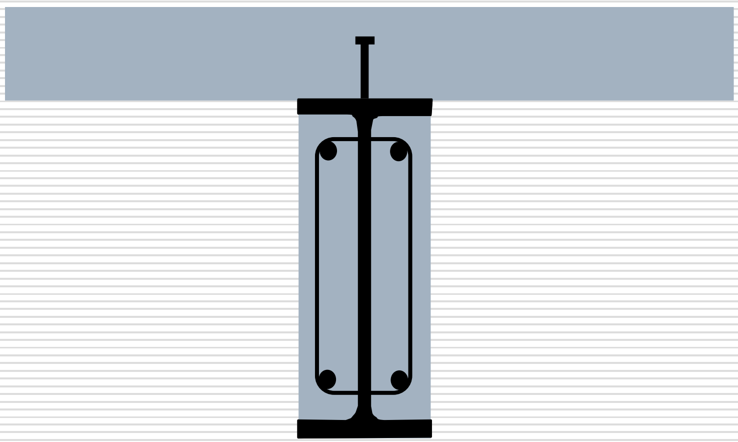
- A variation on this theme is the “Slimflor” system that uses a deep deck laid upon bottom flange extensions to the steel section.



- The connection between concrete and steel provided simply by chemical and friction bond is sufficient to obtain composite action. This allows the overall beam to be relatively shallow and the system forms an obvious rival to the RC concrete flat slab. Figure right illustrates this system.

§ 2.2 Types of beams

- Composite beam behaviour also occurs in situations where concrete is used to encase a steel section, possibly for fire protection. Full encasement has been common for external beams in building frames for many years, however rarely has the benefit of composite behaviour been used.



§ 2.3 Design of beams - generalities

- For each load case the design values for the following effects of actions are applied to members in bending and shall be checked at serviceability limit states (SLS) and ultimate limit states (ULS):
  - For SLS: - vertical deflections
    - cracking of concrete
    - vibrations
  - For ULS: - separate or combined vertical shear force and bending moment
- In the sequence of construction, there are two stages at which the beams will be verified:
  - At construction stage
  - In final stage

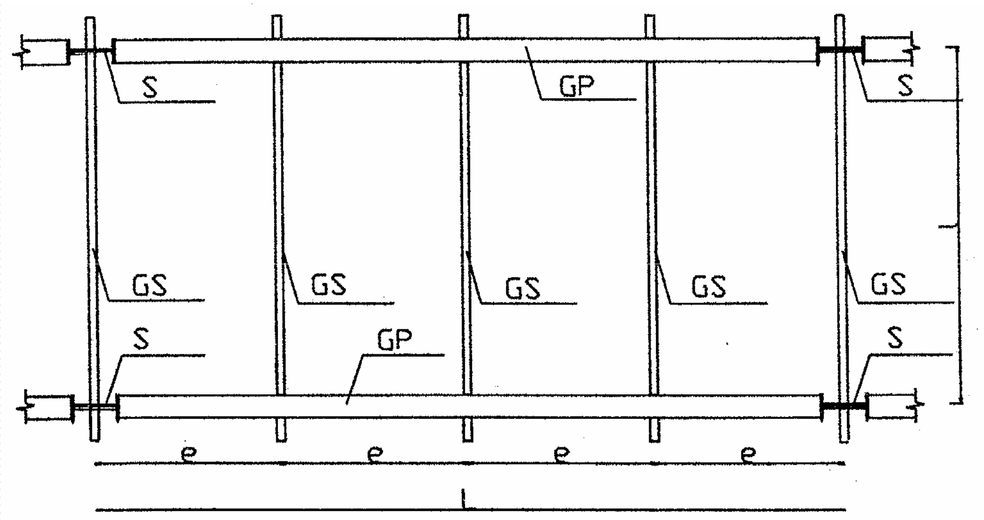
§ 2.3 Design of beams - generalities

- The **construction stage** represents the stage in which the steel part (acting solely) will sustain the wet concrete during casting and curing of concrete. In the construction stage, the steel beam could be:
  - Propped (no checks are necessary)
  - Unpropped (the verification of the steel column beam is necessary in the construction stage)
- The **composite stage** when the concrete is matured, composite beams have to be checked at ULS and SLS, according to Eurocode 4.

Obs: The verifications in the construction stage are made on the steel element only and in consequence the calculus is conducted according to Eurocode 3. This is why, only the verifications in the composite stage are covered by the present course.

§ 2.4 Effective width of composite beams

- Plan view of the floor:

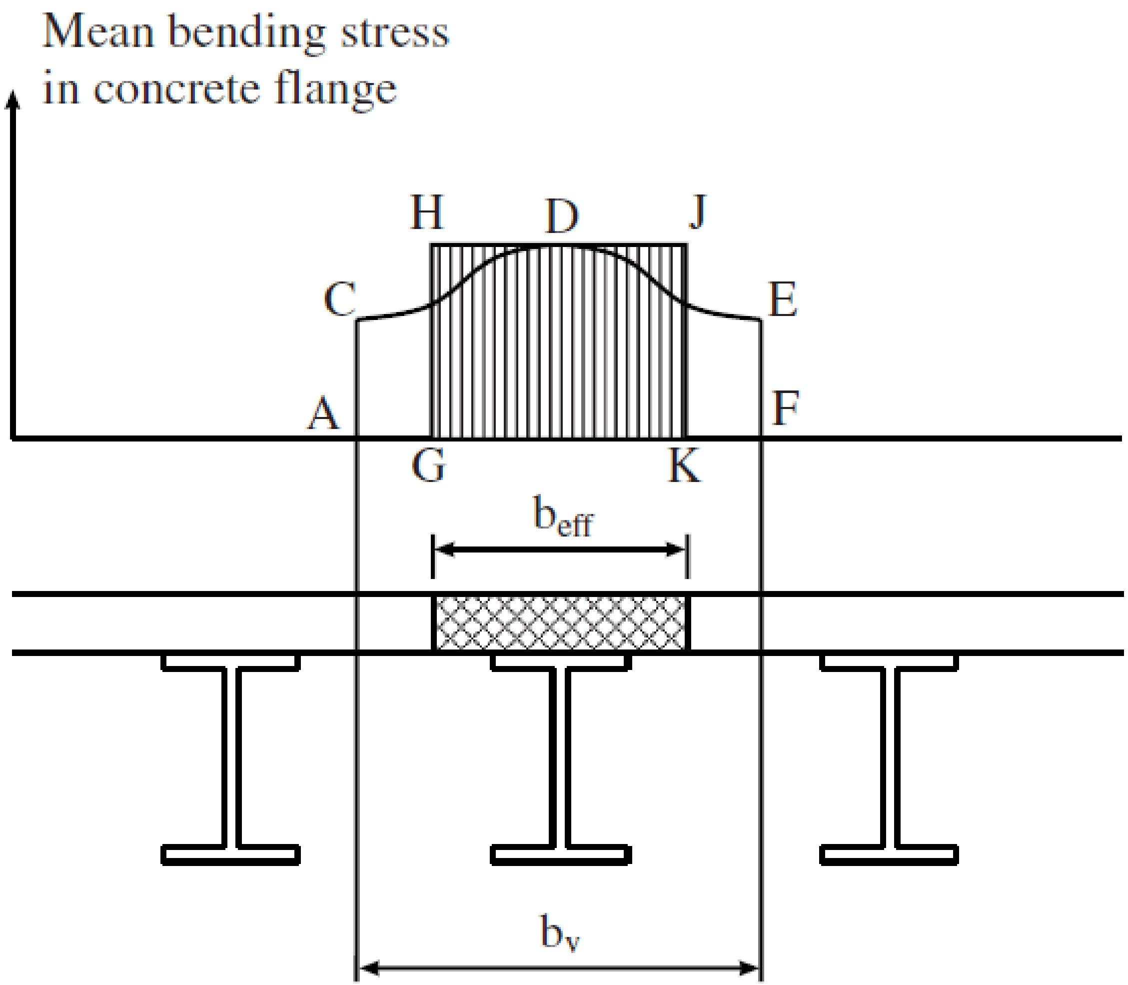


- Considering the fact that the floor beams are distanced at the distance *e*, while the main beams distanced at a *l* distance, the transfer of the shear stress from the steel profile to the concrete slab is less efficient as the distance between the floor beams is greater.

§ 2.4 Effective width of composite beams

- Unlike the reinforced concrete floors, in the case of the composite beams, the effective width b_{eff} of the concrete floor is not always equal to the distance between the steel beams.
- The connection realised at the interface between the two materials practically looses its effect with the distance form the connection.
- Hence, if the distance between the secondary steel beams *e* is important, the concrete slab does not uniformly collaborate to the bending of the composite beam, in consequence the axial stresses being not uniformly distributed. Therefore, the axial stresses in the concrete slab are not uniformly distributed (by so-called “shear lag” effect).
- Conventionally it is considered that steel beam is connected to a concrete slab having the effective width b_{eff}, for which could be admitted that the normal compression efforts are uniformly distributed, see the below figure:

§ 2.4 Effective width of composite beams



Effective width concept

§ 2.4 Effective width of composite beams

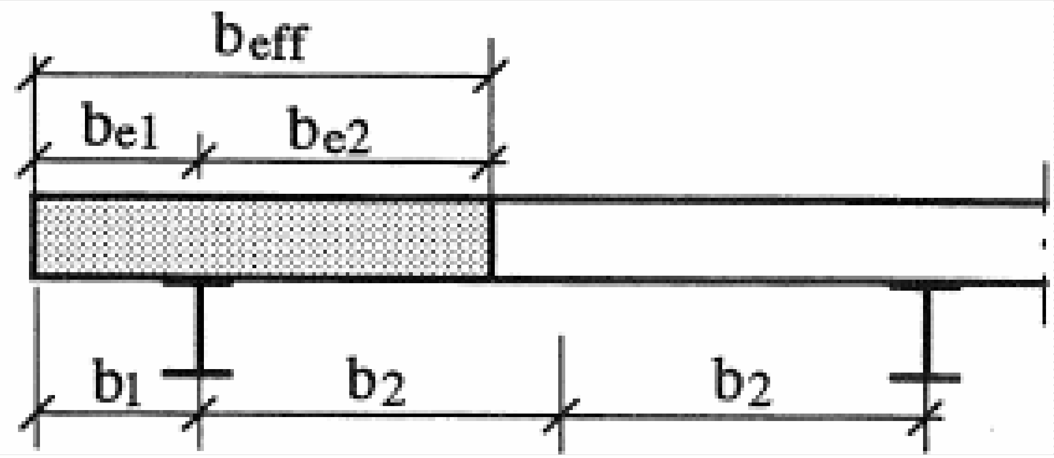
- The research results performed in the purpose of finding some analytical models for the calculation of effective width b_{eff} have proven the following:
  - The effective width b_{eff} is different along the composite beam, its variation depending on the L/e ratio.

    L – the span of the composite beam; e – transversal distance between beams.
  - The effective width b_{eff} depends on the type of loading, namely uniformly distributed or concentrated. It was demonstrated that in the location of concentrated loads the effective width is reduced.
  - The effective width b_{eff} is smaller in the case of partial connection than in the case of a total connection (the interaction of the concrete slab is smaller in the case of partial connection).

§ 2.4 Effective width of composite beams

- For the design of composite beams, the effective width b_{eff} is considered **constant on the entire span**. This value is adopted in the middle of span in the case of simply supported beam, respectively the value on the support for cantilevers or continuous beams on supports.

- The effective width b_{eff} of the composite beam, associated to each supporting steel profile is taken as the sum of the effective breadths b_e of concrete slabs, on each side of the axis of the web of the steel profile.



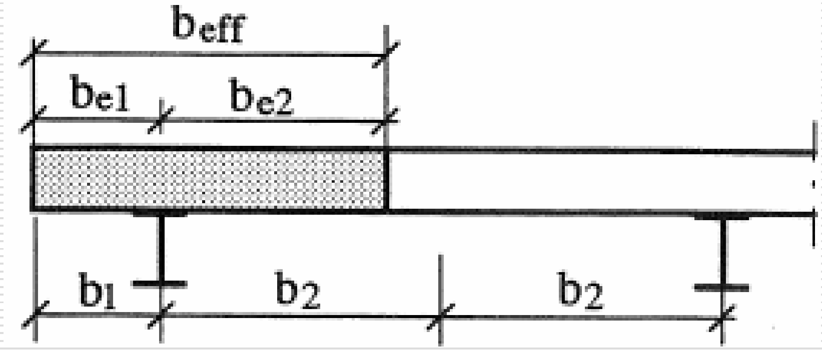
$$\boxed{b_{eff} = b_{e1} + b_{e2}}$$

§ 2.4 Effective width of composite beams

- For the effective breadth, considered on each side of the steel profile – b_e there could be considered the following values:

$$b_{e1} = \text{minimum}\left(\frac{\ell_0}{8}; b_1\right) \qquad b_{e2} = \text{minimum}\left(\frac{\ell_0}{8}; b_2\right)$$

- The real widths b_i from the above formulae are considered as being half of the distance between the considered and the adjacent steel web, measured in the middle of the concrete slab height.

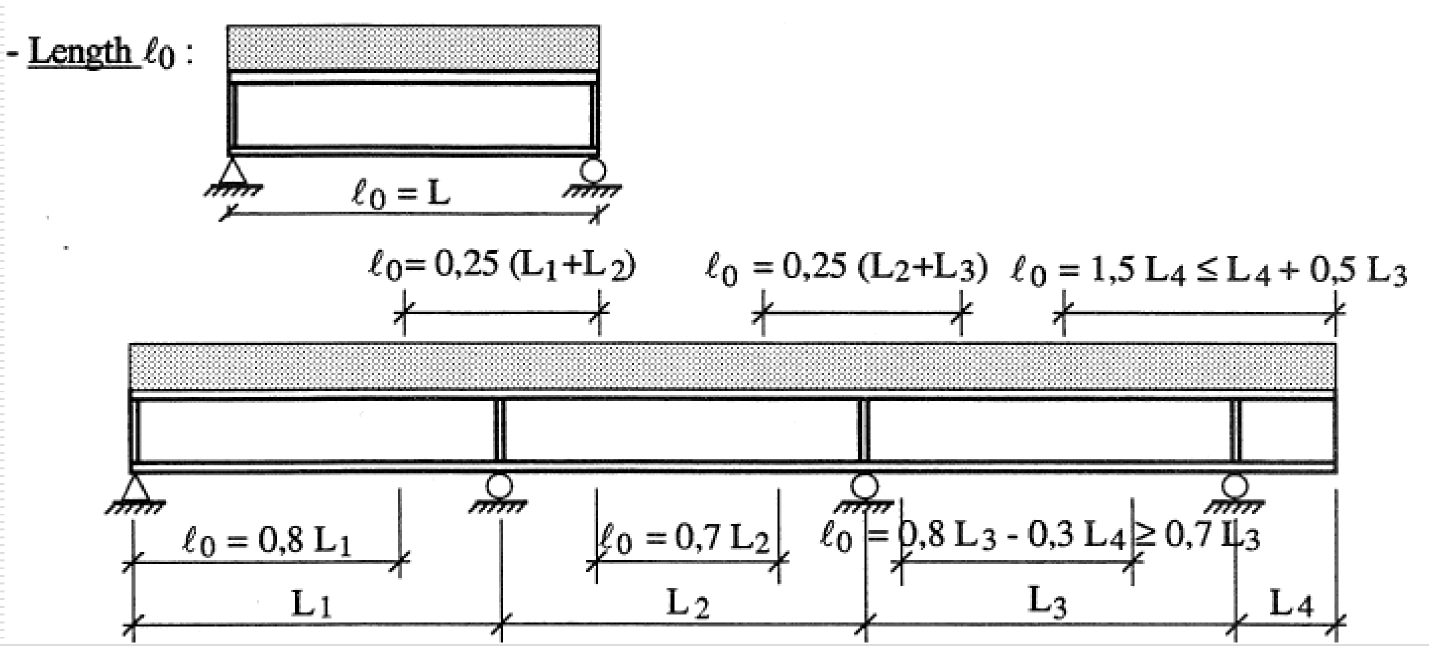


In the case of free edges, this distance is considered between the considered web and the free edge of the slab.

- The length l_0 that is present in the above relations represents the approximate distance between the points of "zero bending moment".

§ 2.4 Effective width of composite beams

- In the case of a simply supported beam, the l_0 length is taken equal to the beam span.
- In the case of continuous beams, the l_0 length could be taken according to the below figure:



§ 2.5 Modular ratio

- In certain circumstances of design the section of composite elements (in both cases of composite columns and beams) is considered as made of a single homogeneous material.
- Conventionally, this material is considered to be the steel. The equivalence is made by the "transformation" of the concrete elements into an "equivalent steel element".
- This is realised by considering the concrete area A_c by an equivalent one A_c/n , where n represents the nominal **modular ratio**, equal to:

  $n = E_a / E'_c$, where

  - E_a elastic modulus for structural steel
  - E'_c effective modulus of concrete

§ 2.5 Modular ratio

- E'_c is computed in function of the type of loading:
  - For short-term effects: $E'_c = E_{cm}$
  - For long-term effects : $E'_c = E_{cm}/3$
  - For other cases: $E'_c = E_{cm}/2$

The values of E_{cm} are given in tables with the concrete characteristics (see Eurocode 2).

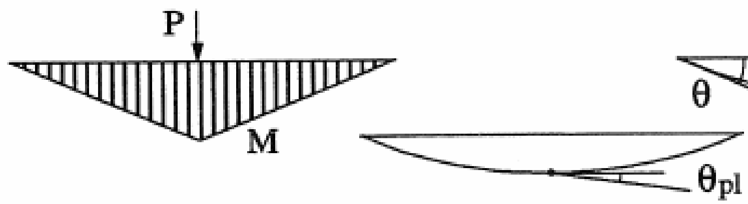
§ 2.6 Classification of composite cross-sections

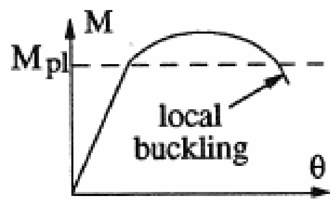
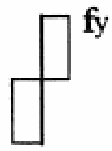
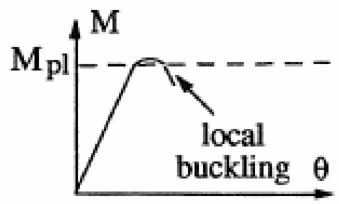
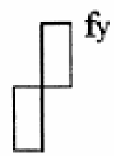
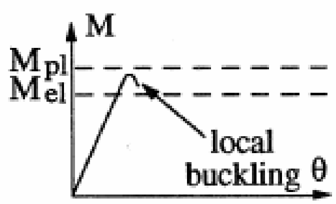
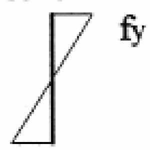
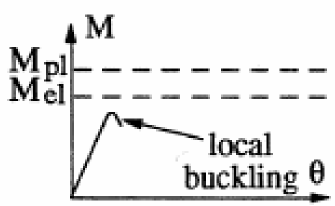
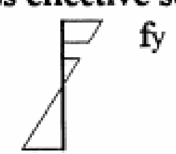
- According to the prescriptions of Eurocode 4, the classification of the steel-concrete is made according to Eurocode 3, section 5.3.2. This section refers to steel cross-sections, but it is applied also in the case of composite cross-sections.
- There are defined four classes of cross-sections:
  - **Class 1**: refers to cross-sections that develop the plastic resistance moment capacity $M_{pl,Rd}$, having a sufficient rotation capacity in order to allow a complete bending moment redistribution in the structure in order to permit the formation of new plastic articulations. In this case, the composite cross-sections are able to form ***plastic articulations***.
  - **Class 2**: refers to cross-sections that may develop the plastic resistance moment capacity $M_{pl,Rd}$, but they posses a **limited rotation capacity**.

§ 2.6 Classification of composite cross-sections

- **Class 3**: refers to cross-sections in which the compression efforts on the extreme fibers of the steel beam may reach the yielding stress, but on which the local buckling is susceptible to hinder the development of the plastic resistance moment $M_{pl,Rd}$.
- **Class 4**: refers to cross-sections compliant to the local buckling phenomenon that appear in the compressed zone of the steel profile, before the attainment of the elastic strength into the extreme fibres. In this case, the compression or bending design strength is determined by taking into account explicitly by the local buckling phenomenon.

- The moment-rotation behaviour curve corresponding to each class of cross-section is given in the picture below :



Class	Behaviour model	Design resistance	Available rotation capacity of plastic hinge	Global analysis of structures
1		**PLASTIC** across full section 	important	plastic or, elastic with redistribution of moments
2		**PLASTIC** across full section 	limited	elastic with redistribution of moments
3		**ELASTIC** across full section 	none	elastic with redistribution of moments
4		**ELASTIC** across effective section 	none	elastic with redistribution of moments

§ 2.6 Classification of composite cross-sections

- The elements of a transversal cross-section (web and flanges) may have different classes. The classes of these elements are computed by the ratios between the height or lateral width and its thickness.
- The class of a transversal cross-section is given by the most unfavourable class of the web or the compressed flange, depending on the case.
- Unlike the steel cross-section, the composite steel-concrete cross-sections are more complex when characterised by their class:
  - The composite cross-sections are generally unsymmetrical (although the steel profile is symmetric) and are not constant on length.
  - A composite cross-section can change its class if the bending moment changes its sign.

Obs: By example, a continuous beam can have a section of class 1 in the zone of hogging moment, but may change the class to 3 or even 4 in the sagging moment (on support).

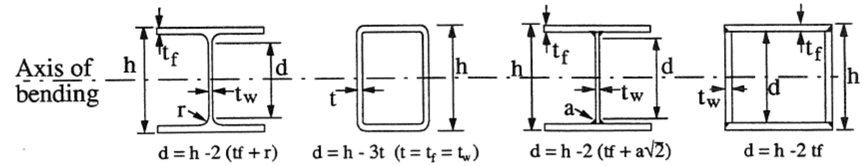
§ 2.6 Classification of composite cross-sections

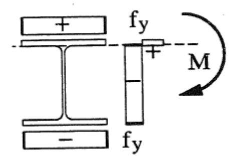
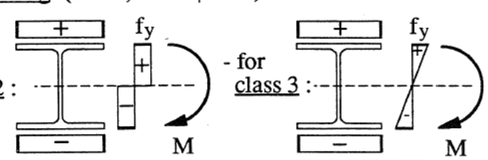
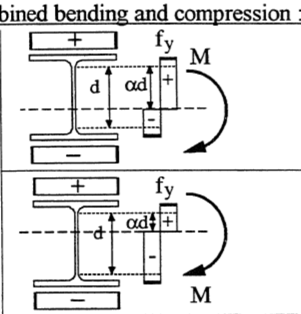
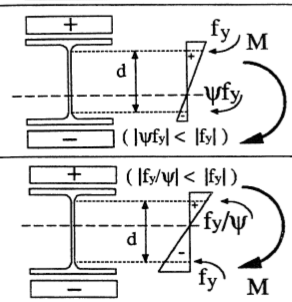
- The compressed concrete, present in the vicinity of the steel profile elements can change (by increasing) the class of the web or flange of the steel profile, by hindering the local buckling phenomenon. This is happening in the conditions in which the link between the steel and concrete elements is very effective.
- By example, in the case in which the steel web is encased in concrete, the concrete cover should be reinforced and connected by mechanical means to the steel profile. Also, it should be capable of hindering the web buckling.
- In practice, there are tables that offer the computed classes for the composite beams, computed for different hot-rolled profiles

§ 2.6 Cla
composite

- The procedure of calculation of the cross-section class in the case of composite beams.

Webs (internal elements perpendicular to axis of bending) :



Stresses distribution on web for different classes	$d / t_w \le$ Class 1	$d / t_w \le$ Class 2	$d / t_w \le$ Class 3
1) Web in compression ($\alpha = \psi = 1$) : 	$33\,\varepsilon$	$38\,\varepsilon$	$42\,\varepsilon$
2) Web in bending ($\alpha = 0{,}5$ and $\psi = -1$) : - for class 1 & 2 : - for class 3 : 	$72\,\varepsilon$	$83\,\varepsilon$	$124\,\varepsilon$
3) Web subjected to combined bending and compression : - For class 1 and 2 : 	if $\alpha > 0{,}5$: $\dfrac{396\varepsilon}{13\alpha - 1}$	if $\alpha > 0{,}5$: $\dfrac{456\varepsilon}{13\alpha - 1}$	
	if $\alpha < 0{,}5$: $\dfrac{36\varepsilon}{\alpha}$	if $\alpha < 0{,}5$: $\dfrac{41{,}5\varepsilon}{\alpha}$	
- For class 3 : ($\lvert\psi f_y\rvert < \lvert f_y\rvert$) 			if $\psi > -1$: $\dfrac{42\varepsilon}{0{,}67 + 0{,}33\psi}$
($\lvert f_y/\psi\rvert < \lvert f_y\rvert$)			if $\psi \le -1$: $62\varepsilon(1-\psi)\sqrt{-\psi}$

$\varepsilon = \sqrt{235 / f_y}$	f_y (N/mm2)	235	275	355	"–" stresses in compression
	ε (if $t_w \le 40$ mm)	1	0,92	0,81	"+" stresses in tension
	ε (if 40 mm $< t_w \le 100$ mm)	1	0,96	0,84	

§ 2.6 C
composit

- Table containing the cross-section classes for different types of composite beams, containing hot-rolled steel profiles (example).

IPE - IPE A - IPE O hot-rolled steel profiles
Classification of flange and web subjected to particular loading

Designation	Class of flange in compression (with web not encased)			Class of web in compression			Class of web in bending (stresses for class 1 and 2)		
	Steel grades			Steel grades			Steel grades		
	S 235	S 275	S 355	S 235	S 275	S 355	S 235	S 275	S 355
IPE 80	1	1	1	1	1	1	1	1	1
IPE 100	1	1	1	1	1	1	1	1	1
IPE 120	1	1	1	1	1	1	1	1	1
IPE 140	1	1	1	1	1	1	1	1	1
IPE 160	1	1	1	1	1	1	1	1	1
IPE 180	1	1	1	1	1	2	1	1	1
IPE 200	1	1	1	1	1	2	1	1	1
IPE 220	1	1	1	1	1	2	1	1	1
IPE 240	1	1	1	1	2	2	1	1	1
IPE 270	1	1	1	2	2	3	1	1	1
IPE 300	1	1	1	2	2	4	1	1	1
IPE 330	1	1	1	2	3	4	1	1	1
IPE 360	1	1	1	2	3	4	1	1	1
IPE 400	1	1	1	3	3	4	1	1	1
IPE 450	1	1	1	3	4	4	1	1	1
IPE 500	1	1	1	3	4	4	1	1	1
IPE 550	1	1	1	4	4	4	1	1	1
IPE 600	1	1	1	4	4	4	1	1	1
IPE 750 x 137	1	1	1	4	4	4	1	1	2
IPE 750 x 147	1	1	1	4	4	4	1	1	1
IPE 750 x 173	1	1	1	4	4	4	1	1	1
IPE 750 x 196	1	1	1	4	4	4	1	1	1
IPE A 80	1	1	1	1	1	1	1	1	1
IPE A 100	1	1	1	1	1	1	1	1	1
IPE A 120	1	1	1	1	1	1	1	1	1
IPE A 140	1	1	1	1	1	2	1	1	1
IPE A 160	1	1	1	1	2	3	1	1	1
IPE A 180	1	1	1	2	2	3	1	1	1
IPE A 200	1	1	1	2	3	4	1	1	1
IPE A 220	1	1	1	2	3	4	1	1	1
IPE A 240	1	1	1	2	3	4	1	1	1
IPE A 270	1	1	1	3	4	4	1	1	1
IPE A 300	1	1	2	3	4	4	1	1	1
IPE A 330	1	1	1	3	4	4	1	1	1
IPE A 360	1	1	1	4	4	4	1	1	1
IPE A 400	1	1	1	4	4	4	1	1	1
IPE A 450	1	1	1	4	4	4	1	1	1
IPE A 500	1	1	1	4	4	4	1	1	1
IPE A 550	1	1	1	4	4	4	1	1	1
IPE A 600	1	1	1	4	4	4	1	1	1
IPE O 180	1	1	1	1	1	1	1	1	1
IPE O 200	1	1	1	1	1	1	1	1	1
IPE O 220	1	1	1	1	1	2	1	1	1
IPE O 240	1	1	1	1	1	2	1	1	1
IPE O 270	1	1	1	1	1	2	1	1	1
IPE O 300	1	1	1	1	2	3	1	1	1
IPE O 330	1	1	1	1	2	3	1	1	1
IPE O 360	1	1	1	1	2	3	1	1	1
IPE O 400	1	1	1	2	2	3	1	1	1
IPE O 450	1	1	1	2	2	4	1	1	1
IPE O 500	1	1	1	2	3	4	1	1	1
IPE O 550	1	1	1	2	3	4	1	1	1
IPE O 600	1	1	1	2	2	4	1	1	1

§ 2.7 Design principles of composite beams

- According to Eurocode 4, the design of composite steel-concrete beams is made only for composite cross-sections that have an axis of symmetry in the web plane.
- In the case of composite structures, the concrete shrinkage is neglected in the ultimate limit state verifications.
- Depending on the sign of the bending at which the beam cross-section is subjected, two cases could be distinguished:
  - When the section is subjected to sagging moments (+), **the concrete slab is compressed** (entirely or in part). In this case, the steel profile is subjected to tension and the reinforcement is ignored in design.
  - When the section is subjected to hogging moments (-), **the concrete slab is under traction** and will be ignored in design. The steel profile is under compression and the composite effect will be taken into account by means of the steel reinforcement existent in the slab.

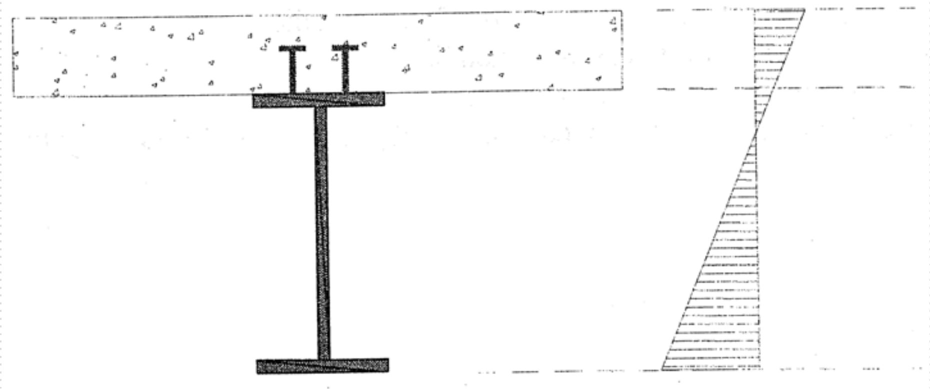
§ 2.7 Computation principles of composite beams

- The bending design resistance of composite beams could be determined by means of a **plastic design** (non-linear analysis), only if the cross-section is of class 1 or 2.
- The linear analysis (**elastic design**) of composite structures could be applied to all cross-section classes of composite beams.

Obs: For the purpose of our course, it is considered that there exists a full-shear (horizontal shear) connection between the steel profile and the concrete slab

§ 2.8 Elastic resistance in bending

- The elastic calculus of composite beams could be done for all the classes of cross-sections (1, 2, 3 and 4).
- The elastic analysis of composite beams could be performed, by considering the following assumptions:
  - the composite beams are realised from a steel profile which is connected on the entire length to a reinforced concrete slab or a composite slab (profiled sheeting and concrete). The connection that exists between the two materials is considered sufficient in order to hinder the slip at the steel-to-concrete interface.



ε diagram

§ 2.8 Elastic resistance in bending

- plane sections remain plane after the deformation;
- steel and concrete are considered elastic materials;
- the concrete under traction is not taken into consideration in the calculation of moment resistance
- the compressive reinforcement is not considered in the computation of moment resistance.

☐ On the basis of the above assumptions, the composite cross-section can be considered to be formed of a single steel homogeneous equivalent material. For this, the geometrical characteristics of the composite cross-section are expressed by means of equivalent geometrical characteristics by using the modular ratio *n* (defined in paragraph 2.4).

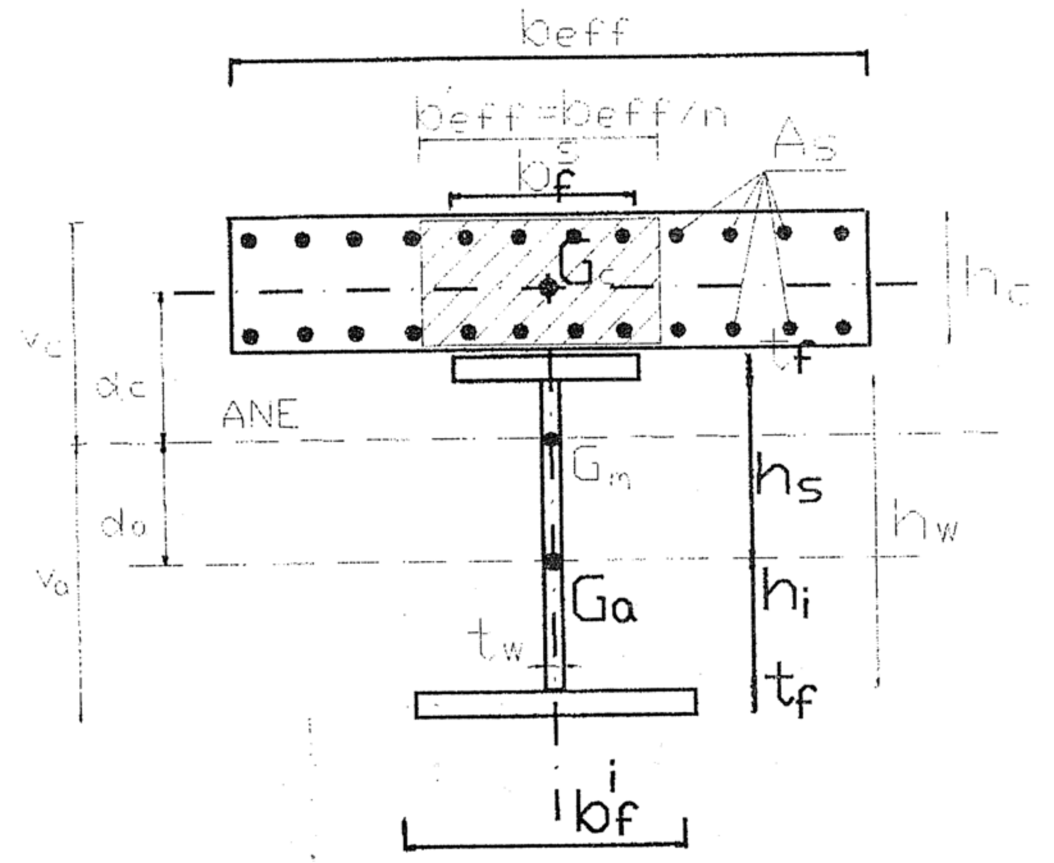
☐ According to the picture below, the equivalent steel area is computed by the following formula:

§ 2.8 Elastic resistance in bending

$$A_1 = A_a + A_s + \frac{A_c}{n} = t_f \cdot b_f^s + t_f \cdot b_f^i + t_w \cdot h_w + A_s + \frac{b_{eff} \cdot h_c}{n}$$

in which:

- A_a represents the area of the steel profile
- A_s is the concrete reinforcement area (neglected if concrete is under compression)
- A_c represents the effective area of the concrete slab
- n is the modular ratio.



§ 2.8 Elastic resistance in bending

- Other equivalent geometrical characteristics as well as the verifications in elastic domain depend on the position of the elastic neutral axis in the cross-section and on bending sign.

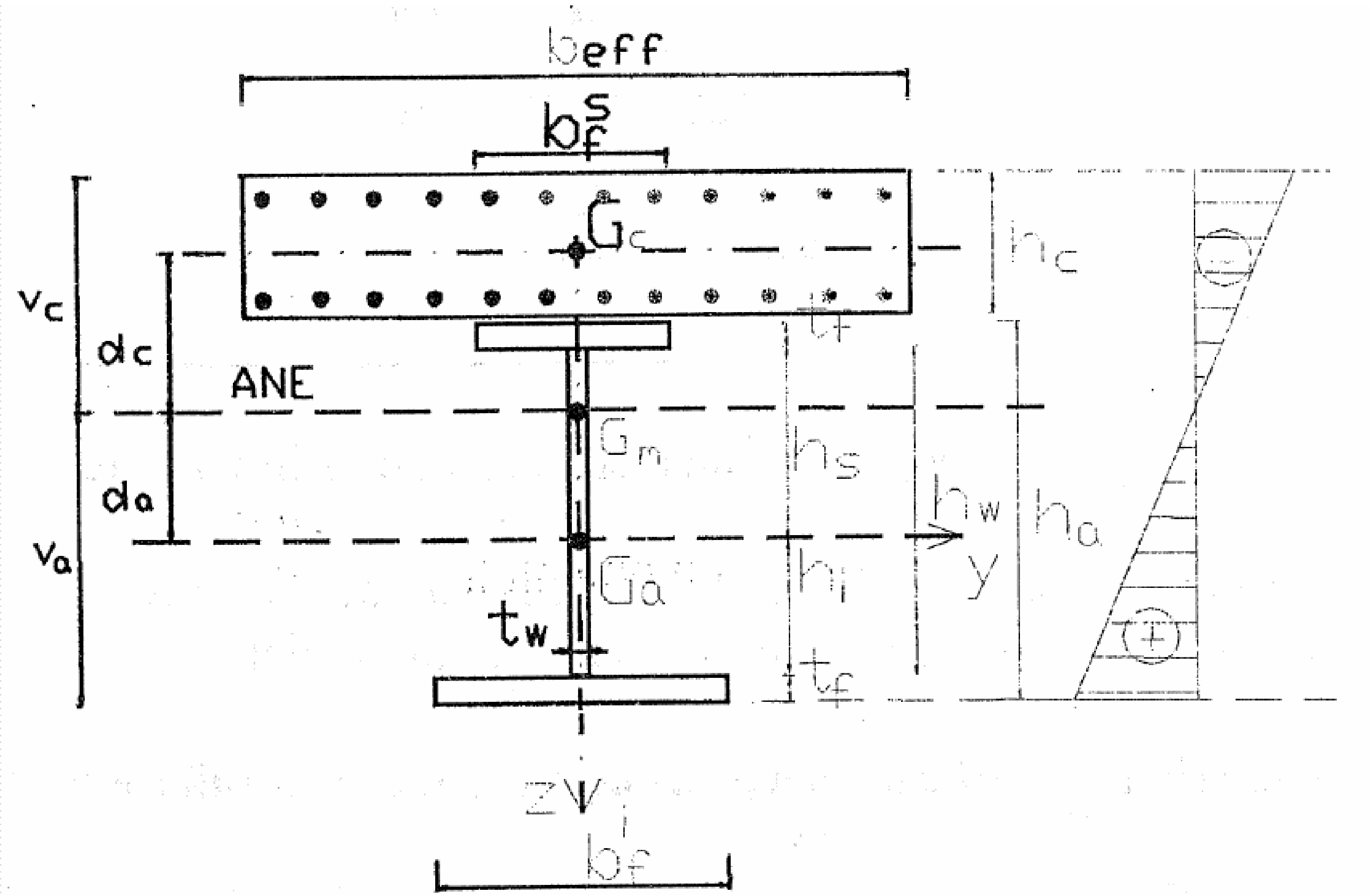
ELASTIC DESIGN OF CROSS-SECTIONS UNDER SAGGING MOMENT

- In this case the superior fibre is compressed (the flexible reinforcement is neglected) while the inferior fibre is in traction.
- There could be distinguished two cases, each of them having different computation procedures:
  - The Elastic Neutral Axis (ENA) is in the concrete slab
  - The Elastic Neutral Axis (ENA) is in the steel profile

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile



§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- The area of the effective concrete section is given by:

$$A_c = b_{eff} \cdot h_c$$

- In consequence, the equivalent steel area of the entire composite section could be written as:

$$A_1 = A_a + \frac{A_c}{n} = t_f \cdot b_f^s + t_f \cdot b_f^i + t_w \cdot h_w + \frac{b_{eff} \cdot h_c}{n}$$

- The position of the centroid of the composite cross-section G_m, computed about the centroid of the concrete slab G_c (by means of the distance d_c), and respectively about the centroid of the steel profile (by means of the distance d_a) is determined through the equality between the static moments of the steel and concrete sections:

$$A_a \cdot d_a = \frac{A_c}{n} \cdot d_c$$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- From the composite beam geometry it could be written:

$$d_a + d_c = h_s + t_f + \frac{h_c}{2} \quad \text{or} \quad d_a = h_s + t_f + \frac{h_c}{2} - d_c$$

- The equality of the static moments could be re-written as:

$$A_a\left(h_s + t_f + \frac{h_c}{2} - d_c\right) = \frac{A_c}{n} \cdot d_c$$

- From which it could be deduced:

$$d_c = \frac{A_a\left(h_s + t_f + \frac{h_c}{2}\right)}{A_a + \frac{A_c}{n}} \quad \text{and} \quad d_a = \frac{\frac{A_c}{n}\left(h_s + t_f + \frac{h_c}{2}\right)}{A_a + \frac{A_c}{n}}$$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- Having the above distances already computed, there could be determined the distances from the centroid of the composite beam G_m to its extreme fibers: v_c (the distance to the superior compressed concrete fibre) respectively v_a (the distance to the inferior steel fibre in traction):

$$v_c = d_c + \frac{h_c}{2} = \frac{A_a\left(h_s + t_f + \frac{h_c}{2}\right)}{A_a + \frac{A_c}{n}} + \frac{h_c}{2}$$

or, by replacing A_c

$$v_c = \frac{A_a\left(h_s + t_f + h_c\right) + \frac{b_{eff} \cdot h_c^2}{2n}}{A_a + \frac{A_c}{n}}$$

$$v_a = d_a + h_i + t_f = \frac{\frac{A_c}{n}\left(h_s + t_f + \frac{h_c}{2}\right)}{A_a + \frac{A_c}{n}} + h_i + t_f$$

or

$$v_a = \frac{\frac{A_c}{n}\left(h_a + \frac{h_c}{2}\right) + A_a\left(h_i + t_f\right)}{A_a + \frac{A_c}{n}}$$

In the above relations h_a represents the height of the steel profile

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- The moment of inertia of the equivalent steel section computed about the principal axis of inertia (y) that passes through the centroid of the composite section G_m is expressed by:

$$I_1 = I_a + \frac{I_c}{n} + A_a \cdot d_a^2 + \frac{A_c}{n} \cdot d_c^2$$

where:

- I_a represents the moment of inertia of the entire steel profile, computed about its own centroid, G_a:

$$I_a = \frac{b_f^s \cdot (t_f)^3}{12} + \left(h_s + \frac{t_f}{2}\right)^2 \cdot b_f^s \cdot t_f + \frac{(h_s)^3 \cdot t_w}{12} + \left(\frac{h_s}{2}\right)^2 \cdot h_s \cdot t_w + \frac{(h_i)^3 \cdot t_w}{12} +$$
$$+ \left(\frac{h_i}{2}\right)^2 \cdot h_i \cdot t_w + \frac{b_f^i \cdot (t_f)^3}{12} + \left(h_i + \frac{t_f}{2}\right)^2 \cdot b_f^i \cdot t_f$$

- I_c represents the moment of inertia of the concrete slab, computed about its own centroid G_c:

$$I_c = \frac{b_{eff} \cdot (h_c)^3}{12}$$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- A_a represents the total area of the steel profile:

$$A_a = t_f \cdot b_f^s + t_f \cdot b_f^i + t_w \cdot h_w$$

- A_c represents the concrete slab area:

$$A_c = b_{eff} \cdot h_c$$

- Having computed the geometrical characteristics of the equivalent steel cross-section, the elastic verification of the cross-section is the usual one, using the classic strength of materials verification formulae.

- The stress checking of cross-section on height is done by:

- In steel: $\sigma = \frac{M}{I_1} z = \frac{M}{W_a} \leq f_y / \gamma_a$ - In concrete: $\sigma = \frac{M}{nI_1} z = \frac{M}{W_c} \leq 0.85 f_{ck} / \gamma_c$

where:

- W_a and W_c are the elastic section moduli for the steel profile and concrete slab respectively, computed by:

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE I: ENA is located in steel profile

- For the inferior fibre of the steel profile: $$W_{ai} = \frac{I_1}{v_a}$$

- For the superior fibre of the steel profile: $$W_{as} = \frac{I_1}{v_c - h_c}$$

- For the inferior fibre of the concrete slab: $$W_{ci} = \frac{nI_1}{v_c - h_c}$$

- For the superior fibre of the concrete slab: $$W_{cs} = \frac{nI_1}{v_c}$$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

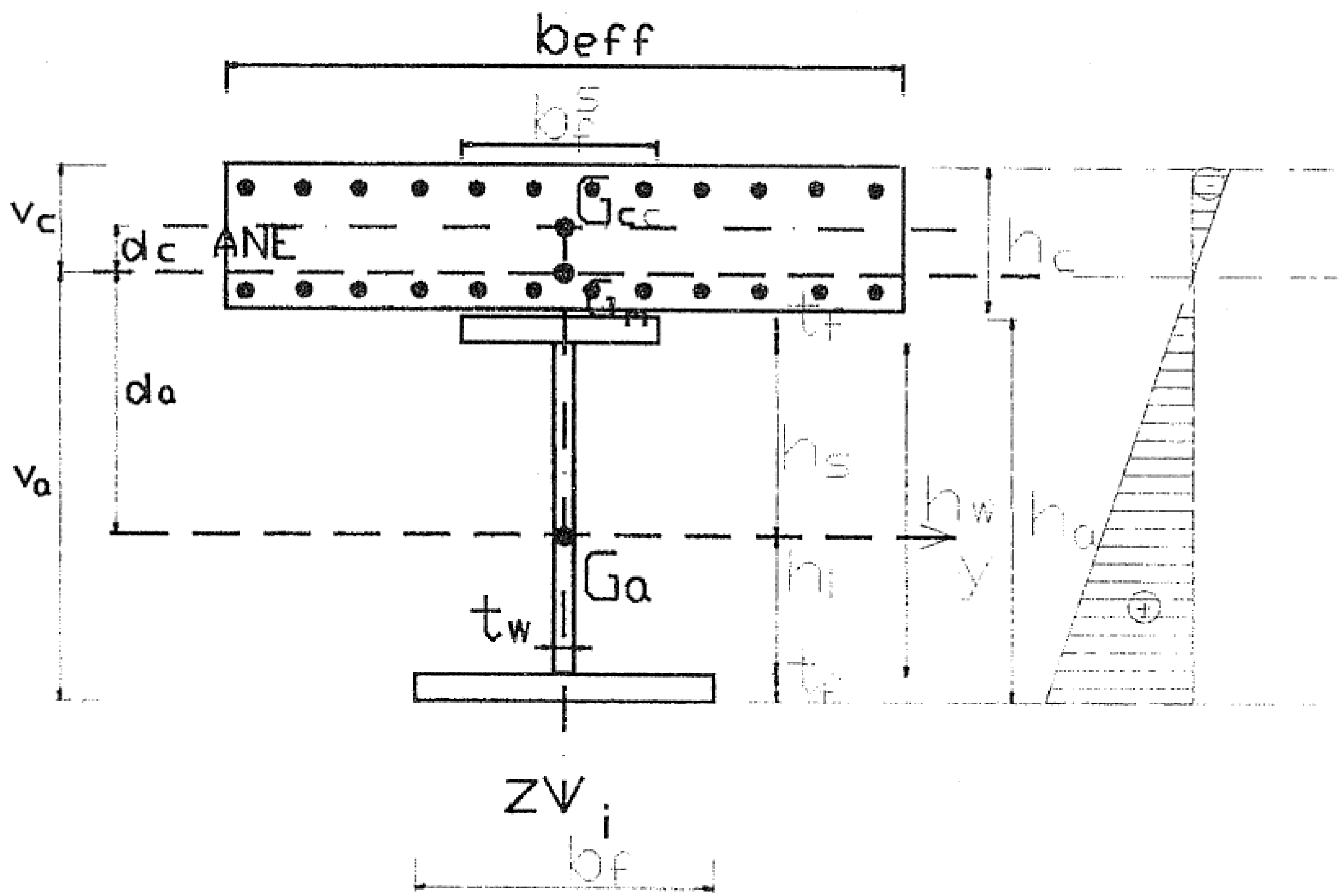
CASE II: ENA is located in concrete slab

- In the case in which the v_c distance, computed as above results to be smaller than the concrete slab height h_c then the elastic neutral axis (ENA) is located into the concrete slab.
- In this case, according to the design assumptions, the concrete slab will be compressed only on the v_c height.
- The efforts diagram for this case is given below.

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE II: ENA is located in concrete slab



§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE II: ENA is located in concrete slab

- The position of the Elastic Neutral Axis is determined identically to the case in which the ENA is located in the steel profile, by the static moments equality of the compressed concrete slab (transformed into steel equivalent section) and the one of the steel profile under traction.
- The area of the compressed concrete (on v_c height) is:

$$A_{cc} = b_{eff} \cdot v_c$$

- The equality of the static moments is written as:

$$\frac{A_{cc}}{n} \cdot d_c = A_a \cdot d_a$$

where:

- A_a represents the steel profile area: $A_a = t_f \cdot b_f^s + t_f \cdot b_f^i + t_w \cdot h_w$
- d_c represents the distance from the centroid of the compressed slab (on v_c height) G_{cc} to the ENA $\quad d_c = \dfrac{v_c}{2}$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE II: ENA is located in concrete slab

- d_a represents the distance from the centroid of the steel profile G_a to ENA: $d_a = h_s + t_f + h_c - v_c$

- Replacing now the d_c and d_a distances, as well as the A_{cc} area into the equation of static equilibrium, the following equation could be written: $$\frac{b_{eff} \cdot v_c^2}{2n} = A_a(h_s + t_f + h_c - v_c)$$

- This equation has only one unknown (v_c) that is found by:

$$v_c = A_a \cdot \frac{n}{b_{eff}} \left[\sqrt{1 + \frac{2b_{eff}}{n \cdot A_a}\left(h_s + t_f + h_c\right)} - 1 \right] < h_c$$

from which it can be deduced: $v_a = h_a + h_c - v_c$

- Form these values results the cross-section characteristics:
  - The equivalent steel area: $A_1 = A_a + \frac{A_{cc}}{n} = A_a + \frac{b_{eff} \cdot v_c}{n}$
  - The equivalent moment of inertia: $I_1 = I_a + A_a \cdot d_a^2 + \frac{I_{cc}}{n} + \frac{A_{cc}}{n} \cdot d_c^2$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE II: ENA is located in concrete slab

where:

- I_a and A_a are the moment of inertia and the area of the steel profile (presented above),
- I_{cc} represents the moment of inertia of the compressed slab:

$$I_{cc} = \frac{b_{eff} \cdot v_c^3}{12}$$

- Replacing these values together with the A_{cc} area and the d_a and d_c distances, it could be computed the moment of inertia of the equivalent steel section about the y-y axis passing through the centroid of the composite section G_m:

$$I_1 = I_a + A_a(h_s + t_f + h_c - v_c)^2 + \frac{b_{eff} \cdot v_c^3}{3n}$$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

CASE II: ENA is located in concrete slab

- The stress checking of cross-section on height is done by:

- In steel: $\sigma = \frac{M}{I_1} z = \frac{M}{W_a} \le f_y / \gamma_a$ - in concrete: $\sigma = \frac{M}{nI_1} z = \frac{M}{W_c} \le 0.85 f_{ck} / \gamma_c$

unde:

- W_a and W_c are the elastic section moduli for the steel profile and concrete slab respectively, computed by:

- For the inferior fibre of the steel profile: $W_{ai} = \frac{I_1}{v_a}$

- For the superior fibre of the steel profile: $W_{as} = \frac{I_1}{h_c - v_c}$

- For the inferior fibre of the concrete slab: $W_{ci} = \frac{nI_1}{h_c - v_c}$

- For the superior fibre of the concrete slab: $W_{cs} = \frac{nI_1}{v_c}$

§ 2.8 Elastic resistance in bending

SECTION UNDER SAGGING MOMENT

- The elastic resistance under sagging bending is computed by:

$$M^{+}_{el.Rd} = \min\left(M^{(1)}_{el.Rd}, M^{(2)}_{el.Rd}\right)$$

where:

- $M^{(1)}_{el,Rd}$ is the elastic resistance evaluated in regard to the inferior fibre of the metallic profile, computed by:

$$M^{(1)}_{el.Rd} = \frac{I_1}{v_a} \cdot \frac{f_y}{\gamma_a}$$

-$M^{(2)}_{el,Rd}$ is the elastic resistance evaluated in regard to the superior concrete fibre, computed by:

$$M^{(2)}_{el.Rd} = \frac{nI_1}{v_c} \cdot \frac{0.85 f_{ck}}{\gamma_c}$$

- The verification of the cross-section is performed by:

$$M^{+}_{Sd} \leq M^{+}_{el.Rd}$$

with M^{+}_{Sd} - the positive bending moment resulted from the static design.

§ 2.8 Elastic resistance in bending

SECTION UNDER HOGGING MOMENT

- In the case of intermediate supports of continuous beams it is necessary the check of these sections too, in the case in which the superior fibres (concrete slab) are under traction, while the inferior fibre of the cross-section (steel profile) is compressed.
- According to the assumptions of elastic design, the concrete is cracked and consequently only the reinforcement will be taken into consideration.
- !!! In the situation of slab in tension, the effective width of the concrete should be reevaluated, according to the prescriptions for hogging moments.
- In this case, the steel area of the active section is:

$$A_2 = A_a + A_s$$

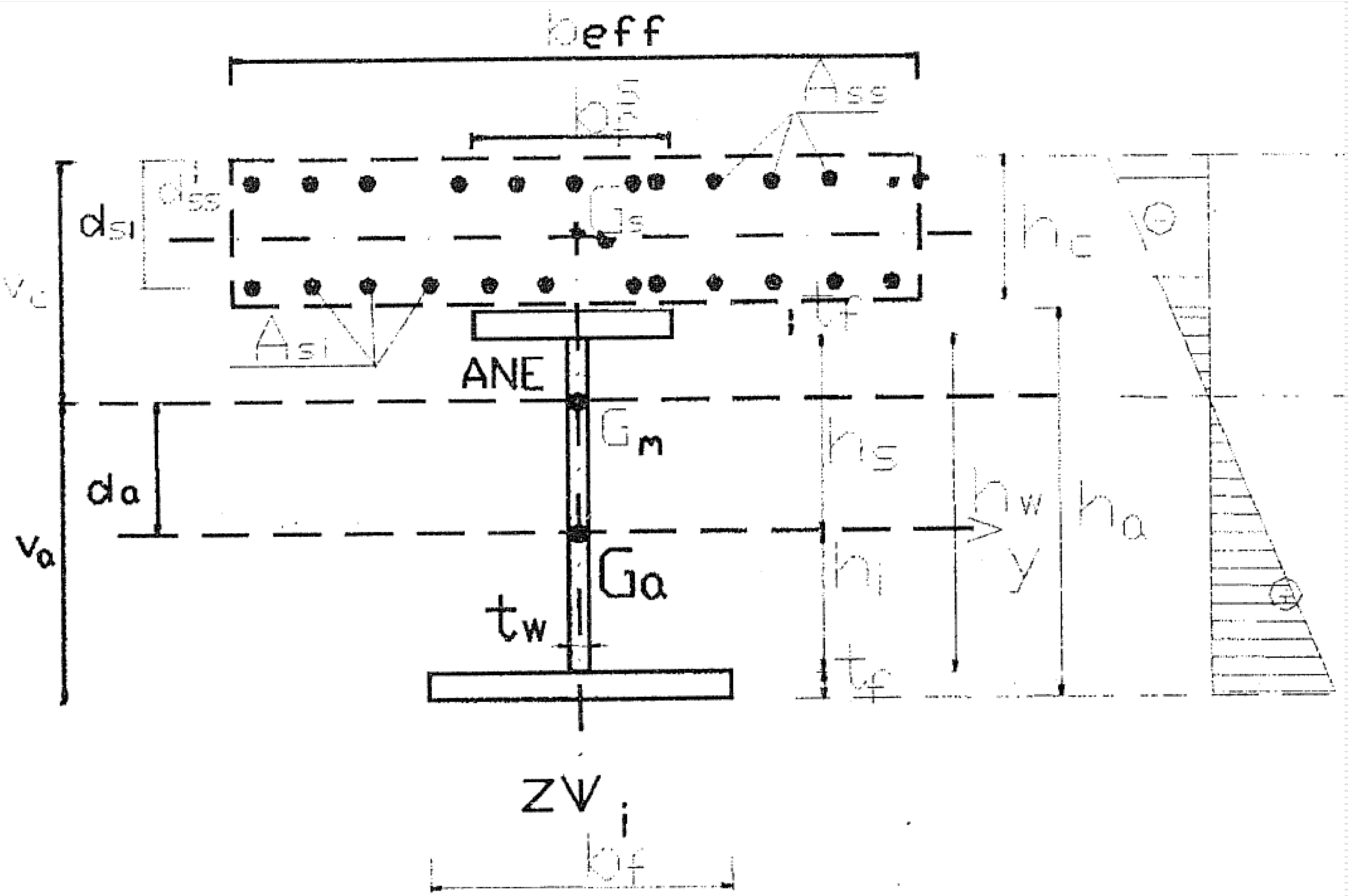
where:

- A_a represents the total area of the steel profile:

$$A_a = t_f \cdot b_f^s + t_f \cdot b_f^i + t_w \cdot h_w$$

§ 2.8 Elastic resistance in bending

SECTION UNDER HOGGING MOMENT



§ 2.8 Elastic resistance in bending

SECTION UNDER HOGGING MOMENT

-A_s represents the area of the flexible reinforcement placed into the concrete slab on the effective width b_{eff}:

$$A_s = A_{si} + A_{ss}$$

- A_{si} and A_{ss} are the reinforcing areas from the inferior and superior fibres of the concrete slab respectively.

- From the equality of the static moments it results:

$$A_a \cdot d_a = A_{ss}(v_c - d_{ss}) + A_{si}(v_c - d_{si})$$

- The distance d_a could be written as: $d_a = h_s + t_f + h_c - v_c$

- Replacing the distance d_a in the formula of equality of the static moments results the distance v_c :

$$v_c = \frac{A_a(h_s + t_f + h_c) + A_{ss} \cdot d_{ss} + A_{si} \cdot d_{si}}{A_2}$$

§ 2.8 Elastic resistance in bending

SECTION UNDER HOGGING MOMENT

In the above formula:

- A_2 represents the area of the active steel cross-section;
- d_{si} and d_{ss} are the distances from the centroids of each row of reinforcement to the extreme superior tensioned fibre of the composite section.

- The distance v_a will be written as: $v_a = h_a + h_c - v_c$
- The moment of inertia of the equivalent steel section is:

$$I_2 = I_a + A_a(h_s + t_f + h_c - v_c)^2 + A_{ss}(v_c - d_{ss})^2 + A_{si}(v_c - d_{si})^2$$

Obs: In the above relation it was ignored the moment of inertia of the flexible reinforcements.

I_a and A_a are the moment of inertia and the cross-section area of the steel profile respectively.

§ 2.8 Elastic resistance in bending

SECTION UNDER HOGGING MOMENT

- The stress checking of cross-section on height is done by:

- In steel: $\sigma = \frac{M}{I_2} z = \frac{M}{W_a} \leq f_y / \gamma_a$

- In reinforcement: $\sigma = \frac{M}{I_2} z = \frac{M}{W_s} \leq f_{sk} / \gamma_s$

unde:

- W_a and W_s are the elastic section moduli for the steel profile and the flexible reinforcements, computed by:

- For the inferior fibre of the steel profile : $W_{ai} = \frac{I_2}{v_a}$

- For the superior fibre of the steel profile : $W_{as} = \frac{I_2}{v_c - h_c}$

- For the inferior reinforcement from slab: $W_{si} = \frac{I_2}{v_c - d_{si}}$

- For the superior reinforcement from slab : $W_{ss} = \frac{I_2}{v_c - d_{ss}}$

§ 2.8 Elastic resistance in bending

HOGGING ELASTIC RESISTANCE

- The elastic resistance under hogging bending is computed by:

$$M^-_{el.Rd} = \min\left(M^{(1)}_{el.Rd}, M^{(3)}_{el.Rd}\right)$$

where:

- $M^{(1)}_{el,Rd}$ is the elastic resistance evaluated in regard to the inferior fibre of the metallic profile, computed by:

$$M^{(1)}_{el.Rd} = \frac{I_2}{v_a} \cdot \frac{f_y}{\gamma_a}$$

-$M^{(3)}_{el,Rd}$ is the elastic resistance evaluated in regard to the superior reinforcement, computed by:

$$M^{(3)}_{el.Rd} = \frac{I_2}{v_c - d_{ss}} \cdot \frac{f_{sk}}{\gamma_s}$$

- The verification of the cross-section is performed by :

$$M^-_{Sd} \le M^-_{el.Rd}$$

with M^-_{Sd} - the negative bending moment resulted from the static design.