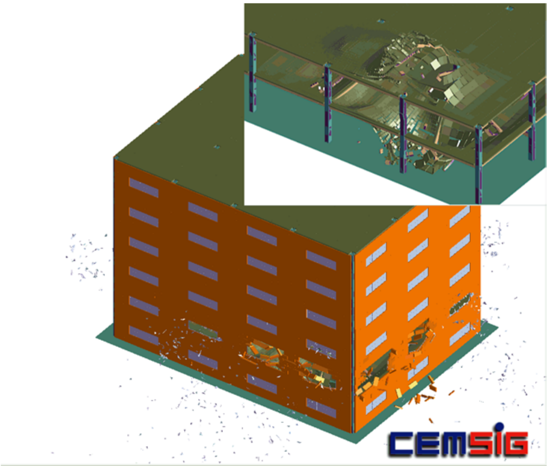
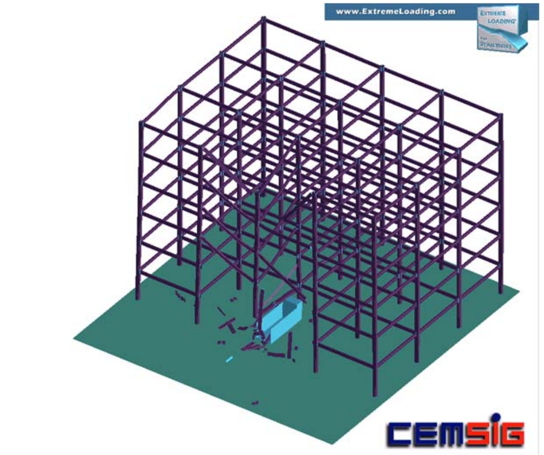
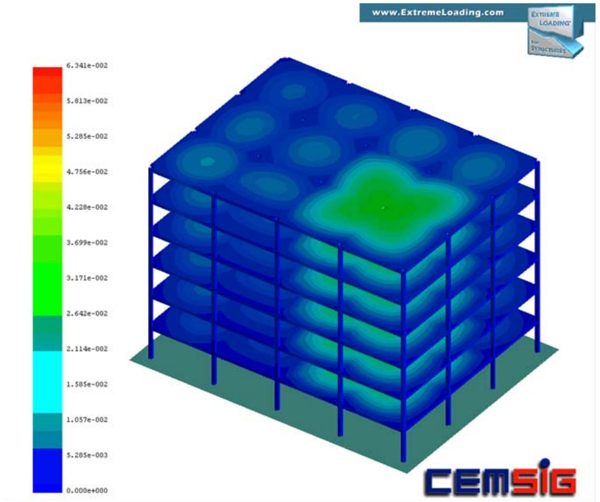
Response to blast







Florea Dinu

Lecture 16-17: 09/04/2014

European Erasmus Mundus Master Course

Sustainable Constructions
under Natural Hazards and Catastrophic Events

520121-1-2011-1-CZ-ERA MUNDUS-EMMC

Universitatea Politehnica Timişoara

Main Text and Reference Materials :

1.Chopra, Anil. – Structural Dynamics

2.Krauthammer, T. – Modern Protective Structures, CRC Press, 2008

3.Smith, P. D. and Hetherington, J. G., “Blast and ballistic loading of structures.”

4.Baker, W. E., et al., “Explosion Hazards and Evaluation.”

5. Donald O. Dusenberry, Handbook for blast-resistant design of buildings

6. Biggs, J.M. (1964), “Introduction to Structural Dynamics”, McGraw-Hill, New York.

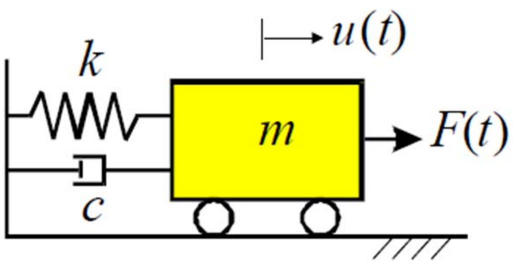
7. T. Ngo, P. Mendis, A. Gupta & J. Ramsay, Blast Loading and Blast Effects on Structures – An Overview, EJSE Special Issue: Loading on Structures (2007)

Structural response to blast loading

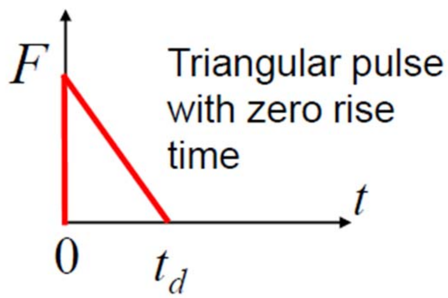
- Complexity in analyzing the dynamic response of blast-loaded structures:
  - uncertainties of blast load calculations
  - time-dependent deformations
  - effect of high strain rates
  - non-linear inelastic material behavior
- To simplify the analysis, a number of assumptions related to the response of structures and the loads has been proposed and widely accepted:
  - Elastic SDOF Systems
  - Elasto-Plastic SDOF Systems
- Blast loading effects:
  - Global structural behavior
  - Localised structural behavior
  - Pressure-Impulse (P-I) Diagrams

Elastic SDOF systems

- The simplest discretization of transient problems is by means of the SDOF approach
- The SDOF system may represent a structure or a structural component, the response parameter of interest and how blast load is applied
- The actual structure can be replaced by an equivalent system of one concentrated mass and one weightless spring representing the resistance of the structure against deformation.
- The blast load can be idealized as a triangular pulse



SDOF system



Blast loading

- Equation of motion :

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F(t)$$

- Solution to equation of motion

$$u(t) = u_h(t) + u_p(t)$$

Homogeneous Particular

where:

$u_h(t)$ satisfies $$m\ddot{u}_h(t) + c\dot{u}_h(t) + ku_h(t) = 0$$

with initial conditions - Free Vibration

$u_p(t)$ satisfies $$m\ddot{u}_p(t) + c\dot{u}_p(t) + ku_p(t) = F(t)$$

with given forces - Forced Vibration

- The equation of motion of the un-damped (c = 0) elastic SDOF system for a time ranging from 0 to the positive phase duration, t_d, is given by:

$$m\ddot{u}(t) + ku(t) = F(t)$$

where the forcing function is given as:

$$\begin{cases} F(t) = F(1 - t/t_d) & t \leq t_d \\ F(t) = 0 & t \geq t_d \end{cases}$$

- For $t \leq t_d$

Particular solution: $u_p(t) = \frac{F}{k}(1 - \frac{t}{t_d})$

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F}{k}(1 - \frac{t}{t_d})$$

$$\omega_n = \sqrt{k/m}$$

ω = natural frequency of vibration

Satisfying initial condition:

$$u(0) = 0\ ,\ \dot{u}(0) = 0$$

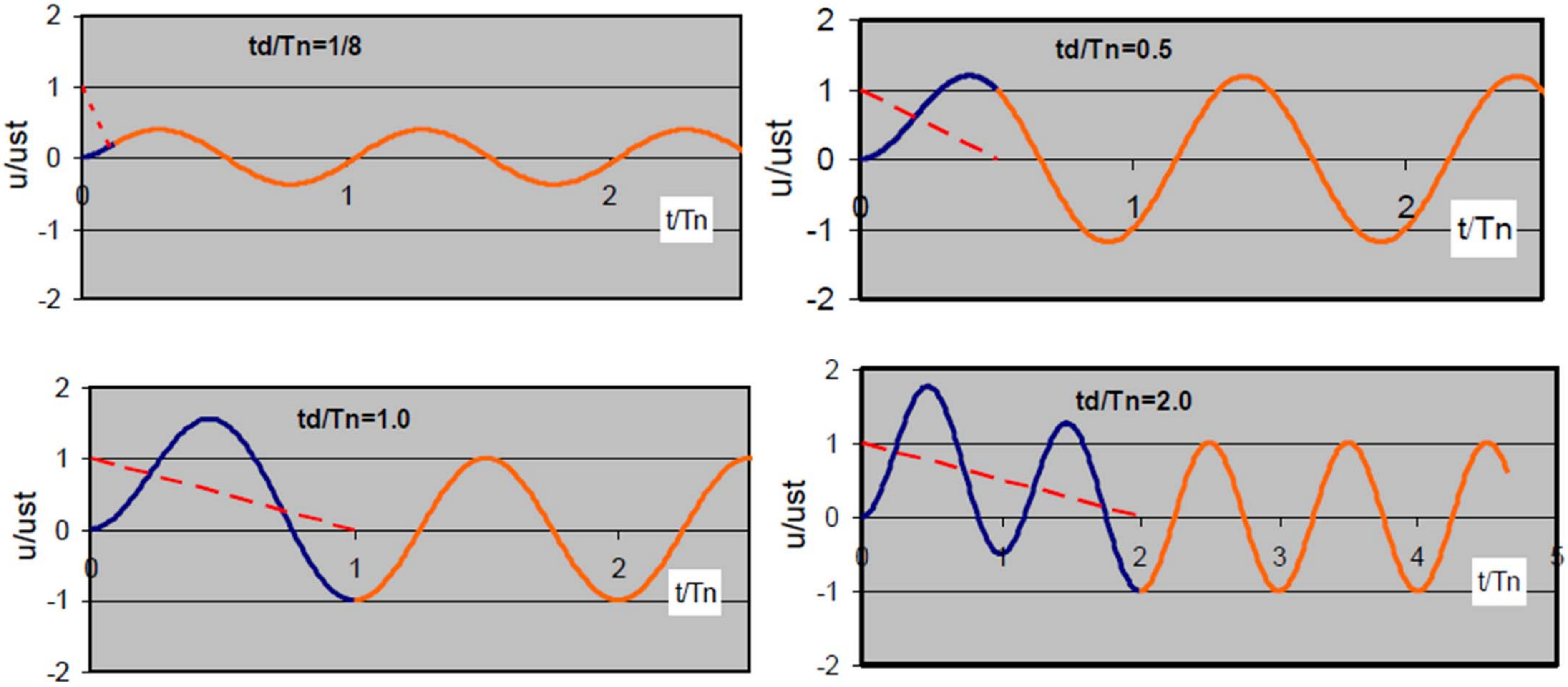
==> Solve for A, B, then obtain u(t) for $t \leq t_d$

$$u(0) = 0 \Rightarrow A = -\frac{F}{k} \quad and \quad \dot{u}(0) = 0 \Rightarrow B = \frac{F}{k}\frac{1}{\omega_n t_d}$$

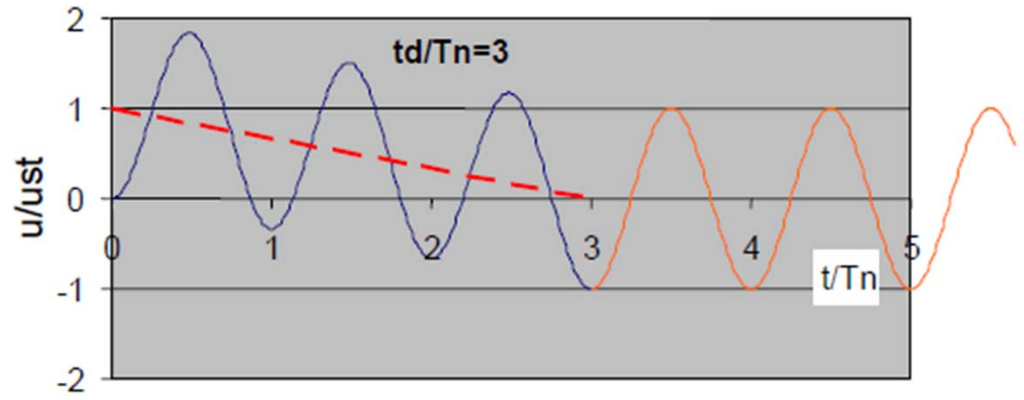
$$u(t) = \frac{F}{k}\left(1 - cos\,\omega_n t\right) + \frac{F}{k}\left(\frac{sin\,\omega_n t}{\omega_n t_d} - \frac{t}{t_d}\right)$$

Subsequently using $u(t_d)\ ,\ \dot{u}(t_d)$ as initial condition for free vibration starting from $(t - t_d)$ to get the *u(t)* for $t \geq t_d$

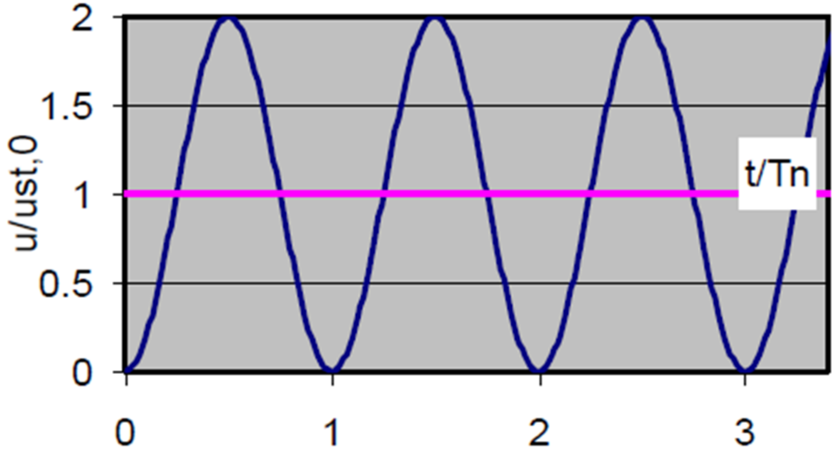
Example response histories



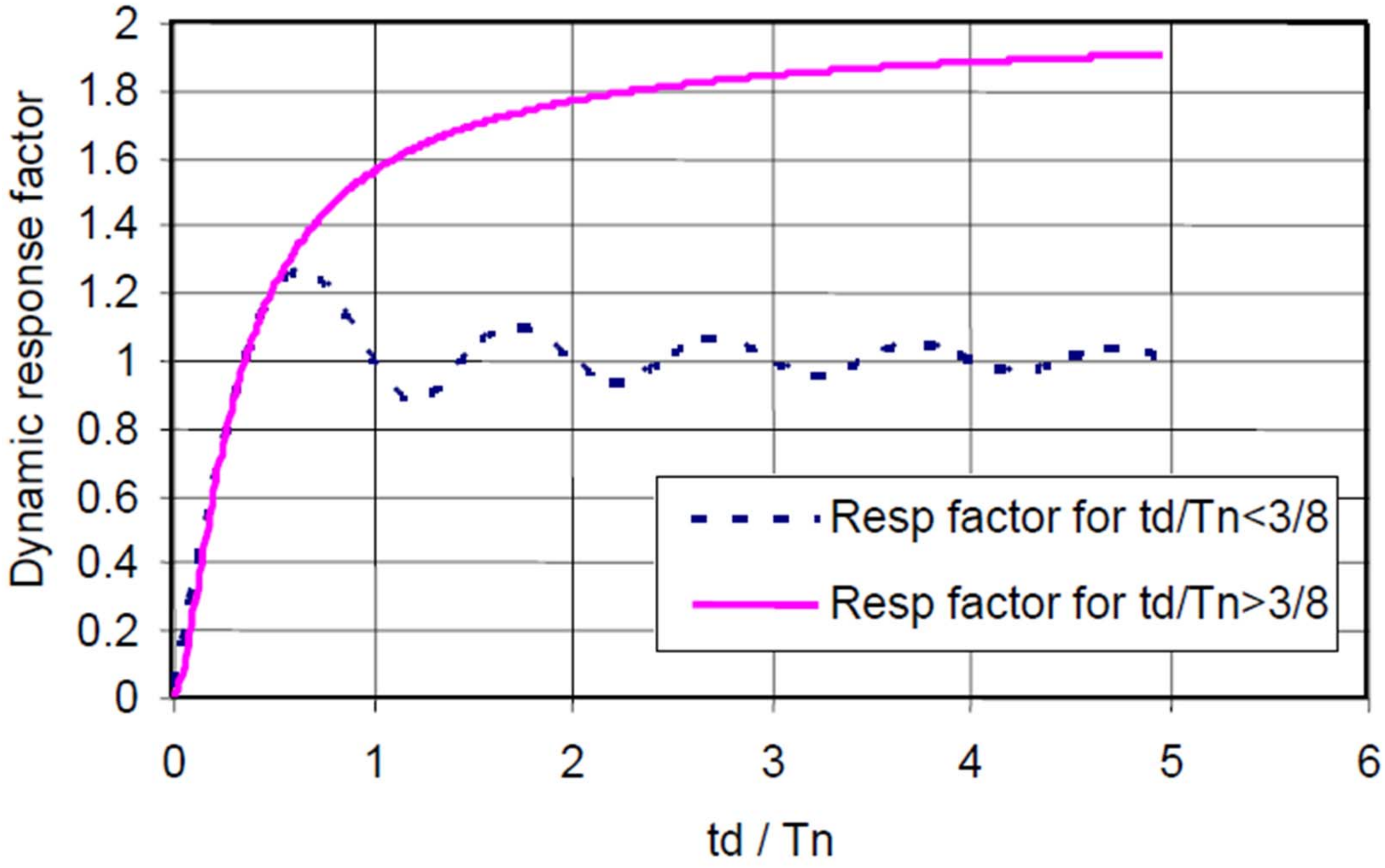
If the ratio t/T_n becomes greater, more oscillations occur during the presence of the forcing function.



$t_d/T_n \rightarrow \infty$ (step force)

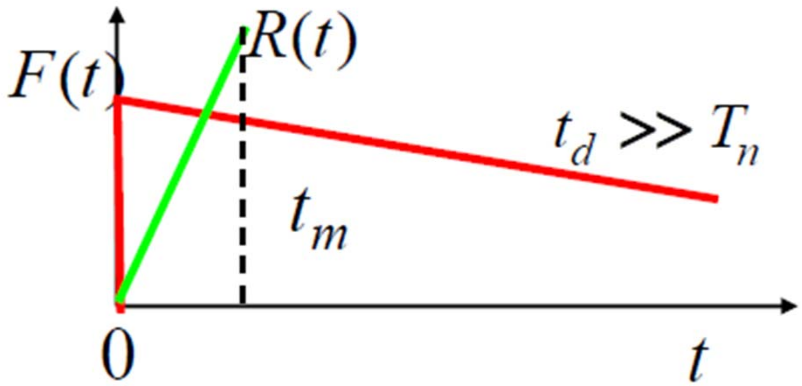


The dynamic load factor

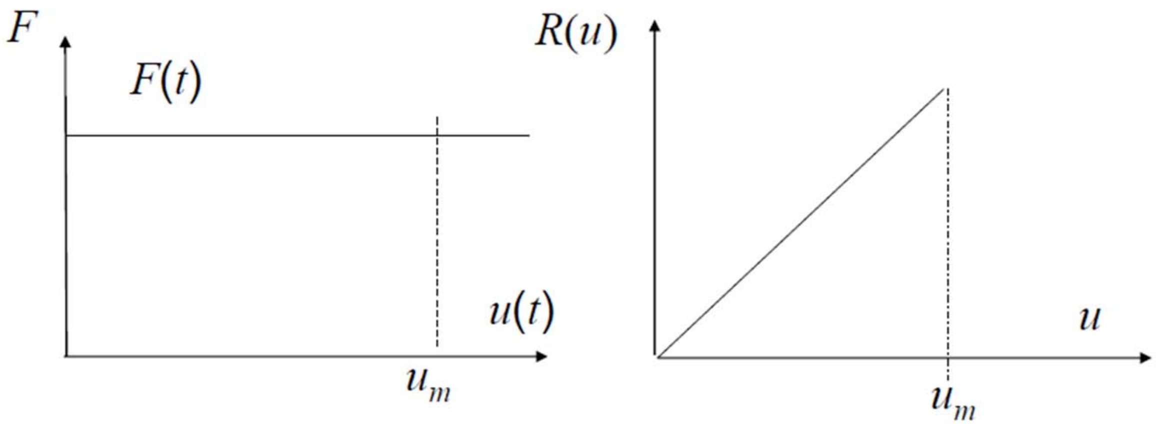


- The spectrum curve can be constructed in a simpler way by looking at two extreme situations:

1) Quasi-static or pressure loading: long t_d, short T_n



SDOF reaches u_m before load has any significant decay,
F(t) ≈ F



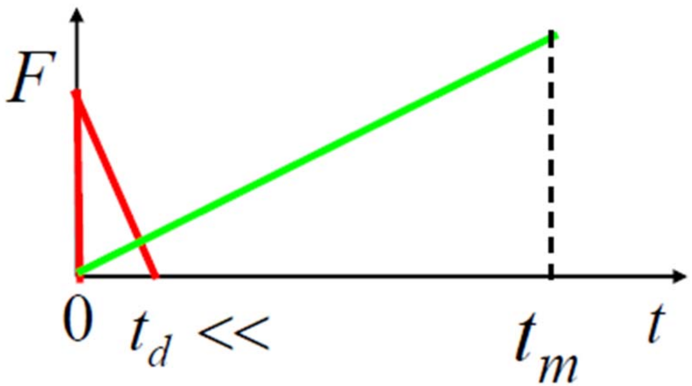
Here consider system energy:

$$Fu_m = \frac{1}{2} k u_m^2$$

$$\frac{u_m}{F/k} = 2$$ or $$DLF = \frac{u_m}{u_{st}} = 2$$

Quasi-static asymptote

2) Impulsive loading: very short t_d, long T_n



The load is applied so quickly even before the SDOF system has any movement. Response treated as free vibration with initial velocity due to impulse. Strain energy stored is the same as previous case.

$$I = \frac{1}{2} F t_d = m\dot{u}_0 \qquad \dot{u}_0 = \frac{I}{m}$$

Kinetic energy: $$KE = \frac{1}{2} m\dot{u}_0^2 = \frac{I^2}{2m}$$

Equating kinetic energy with stored strain energy:

$$\frac{I^2}{2m} = \frac{1}{2} k u_m^2 \Rightarrow u_m = \frac{I}{\sqrt{km}}$$

$$DLF = \frac{u_m}{F/k} = \frac{I}{\sqrt{km}(F/k)}$$ but $$I = \frac{1}{2} F t_d = m\dot{u}_0$$

$$DLF = \frac{1/2 F t_d}{\sqrt{km}(F/k)} = \frac{1}{2}\omega_n t_d$$

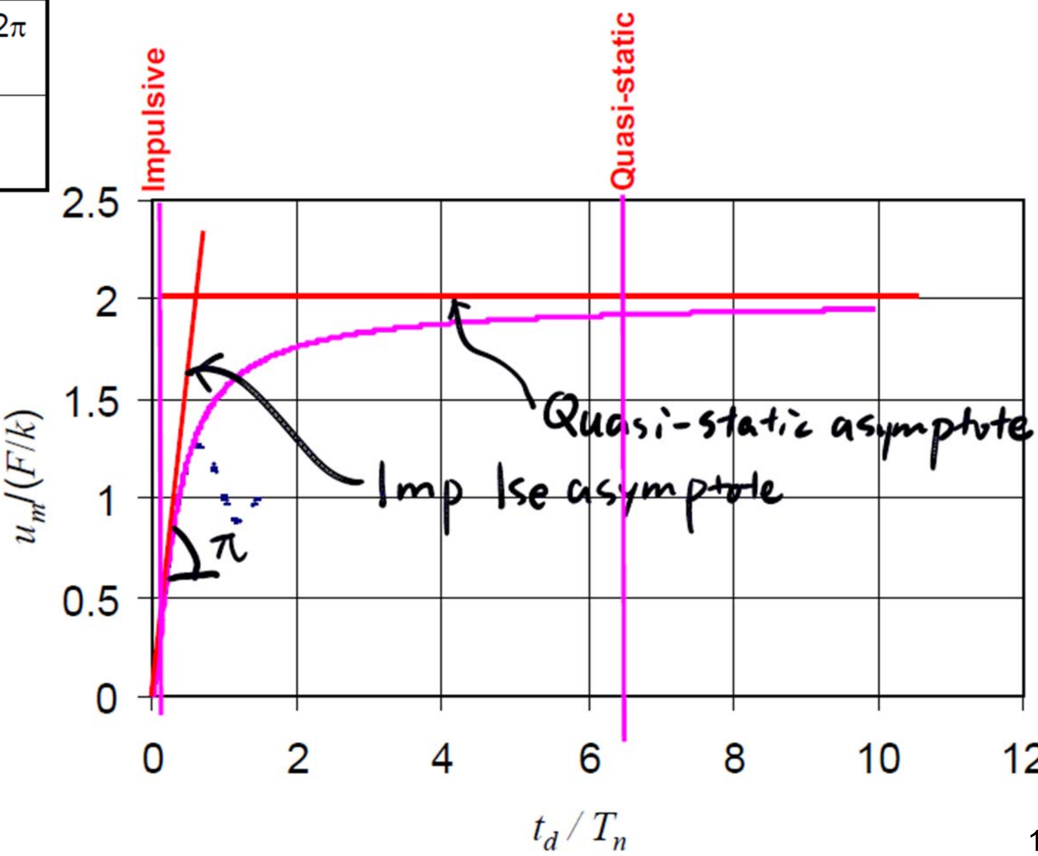
$$DLF = \pi \frac{t_d}{T_n}$$

Impulsive asymptote

Summary of three regimes

Boundaries of three regimes can be specified in terms of the product ωt_d or t_d/T_n as below:

Impulsive loading	$0.4 > \omega t_d$	$0.02\pi > t_d/T_n$
Dynamic loading	$0.4 < \omega t_d < 40$	$0.02\pi < t_d/T_n < 2\pi$
Quasi-static loading	$40 < \omega t_d$	$2\pi < t_d/T_n$



Elasto-plastic SDOF systems

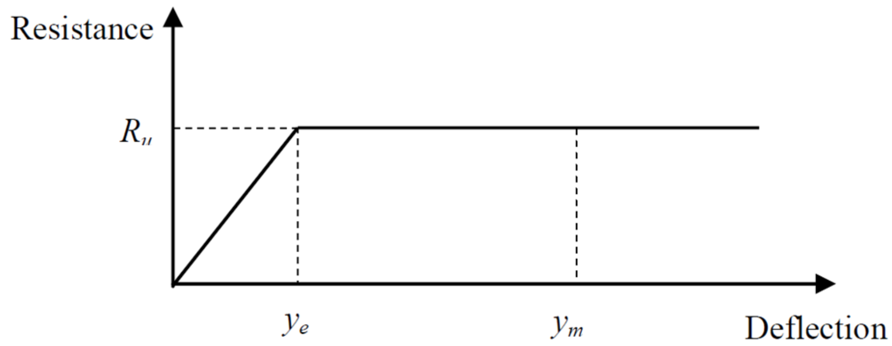
- Structural elements are expected to undergo large inelastic deformation under blast load or high velocity impact.
- Exact analysis of dynamic response is then only possible by step-by-step numerical solution requiring nonlinear dynamic finite-element software.
- However, the degree of uncertainty in both the determination of the loading and the interpretation of acceptability of the resulting deformation is such that solution of a postulated equivalent ideal elasto-plastic SDOF system is commonly used (Biggs, 1964).
- Interpretation is based on the required ductility factor $\mu = y_m/y_e$.

- For example, uniform simply supported beam has first mode shape and the equivalent mass:

$$\varphi(x) = \sin \pi x/L$$

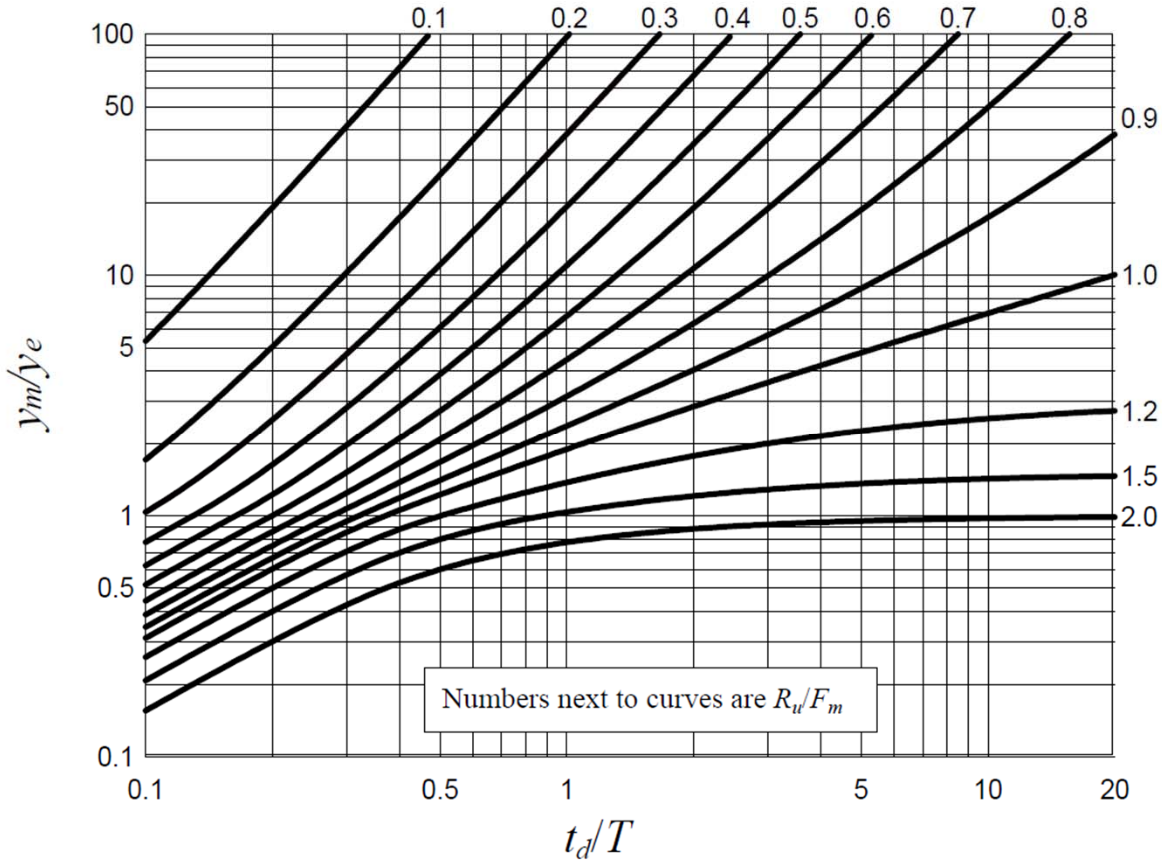
$$M = (1/2)mL$$

where L is the span of the beam and m is mass per unit length.



Simplified resistance function of an elasto-plastic SDOF system

- The equivalent force corresponding to a uniformly distributed load of intensity p is $F = (2/\pi)pL$.
- The response of the ideal bilinear elasto-plastic system can be evaluated in closed form for the triangular load pulse comprising rapid rise and linear decay, with maximum value F_m and duration t_d.
- The result for the maximum displacement is generally presented in chart form as a family of curves for selected values of R_u/F_m showing the required ductility μ as a function of t_d/T, in which R_u is the structural resistance of the beam and T is the natural period



Maximum response of elasto-plastic SDF system to a triangular load

Blast loading effects

- Blast loading effects on structural members may produce both local and global responses associated with different failure modes
- The type of structural response depends mainly on:
  - the loading rate
  - the orientation of the target with respect to the direction of the blast wave propagation
  - boundary conditions
- Failure modes associated with global response: flexure, direct shear or punching shear
- Failure modes associated with local response (close-in effects): localized breaching and spalling

Global structural behavior

- The global response of structural elements is generally a consequence of transverse (out-of-plane) loads with long exposure time (quasi-static loading):
  - global membrane (bending)
  - shear responses:
    - diagonal tension,
    - diagonal compression
    - punching shear
    - direct (dynamic) shear

Have relatively minor structural effect in case of blast loading and can be neglected

The high shear stresses may lead to direct global shear failure and may occur prior to any occurrence of significant bending deformations.

Local structural behavior

- The close-in effect of explosion may cause localized shear (localized punching - or breaching and spalling) or flexural failure in the closest structural elements.
- Breaching failures are typically accompanied by spalling and scabbing of concrete covers as well as fragments and debris

Breaching failure due to a close-in explosion of 6000kg TNT equivalent

Pressure-Impulse (P-I) Diagrams (Iso-damage curves)

- The pressure-impulse (*P-I*) diagram is an easy way to mathematically relate a specific damage level to a combination of blast pressures and impulses imposes on a particular structural element
- There are *P-I* diagrams that concern with human response to blast as well. In this case, there are three categories of blast-induced injury, namely: primary, secondary, and tertiary injury

From SDOF to P-I diagram

- Modify the axis of diagram on slide 15 to become normalized force (pressure) vs. normalized impulse (force x duration) w.r.t displacement

Step 1: inverting vertical axis and scale to

$$\frac{u_m}{F/k} = 2 \qquad y = \frac{2F}{ku_m} \qquad \frac{\text{load (pressure)}}{\text{max. resistance}}$$

Hence quasi-static asymptote becomes:

$$y = \frac{2F}{ku_m} = 1$$

Step 2: multiply abscissa (duration) by the new ordinate (already force measure) and scaling:

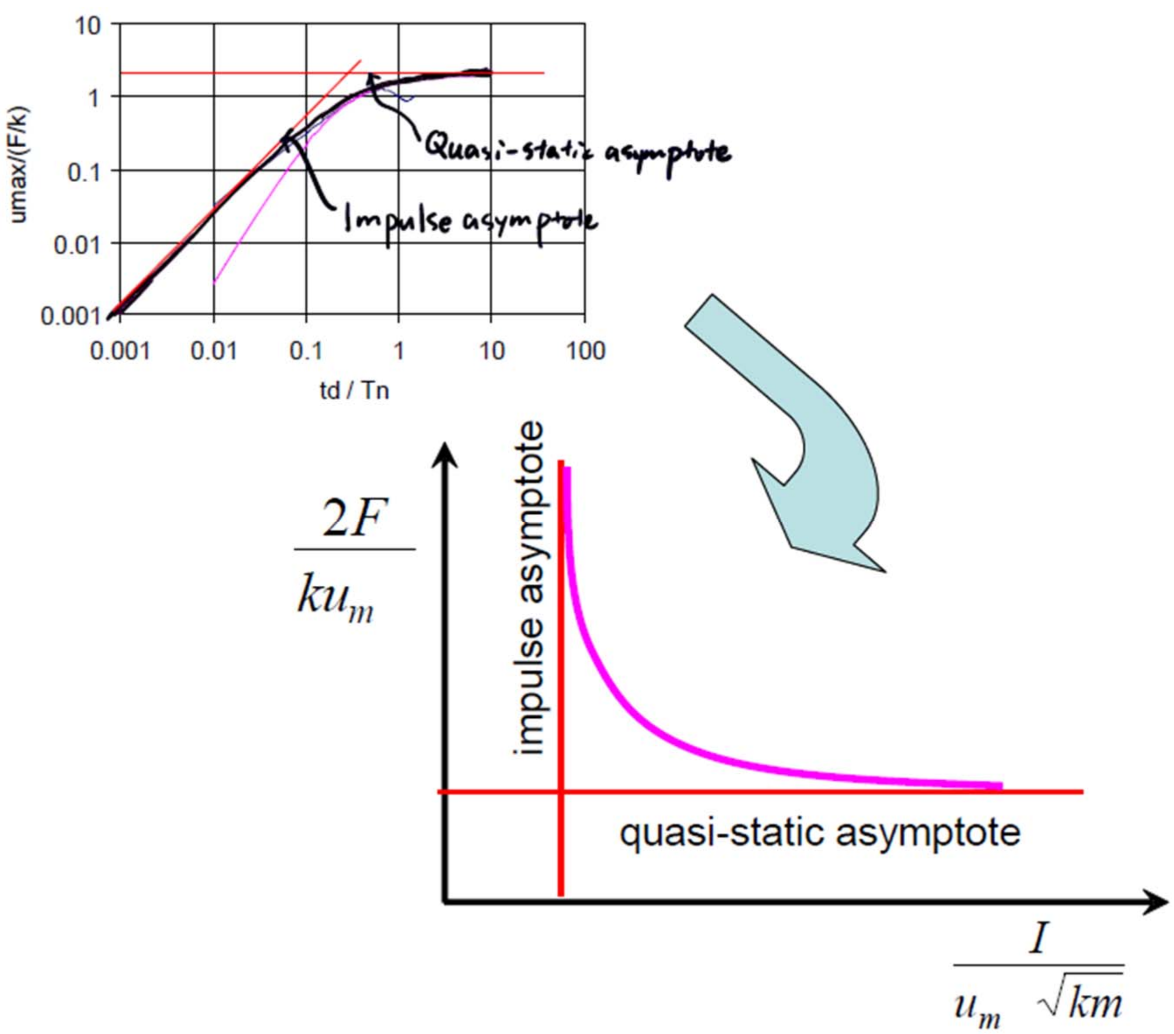
$$x = \pi \frac{t_d}{T_n}\left(\frac{F}{ku_m}\right) = \frac{1}{2}\omega_n t_d \left(\frac{F/k}{u_m}\right) = \frac{1/2Ft_d}{u_m\sqrt{km}}$$

$$x = \frac{I}{u_m\sqrt{km}}$$

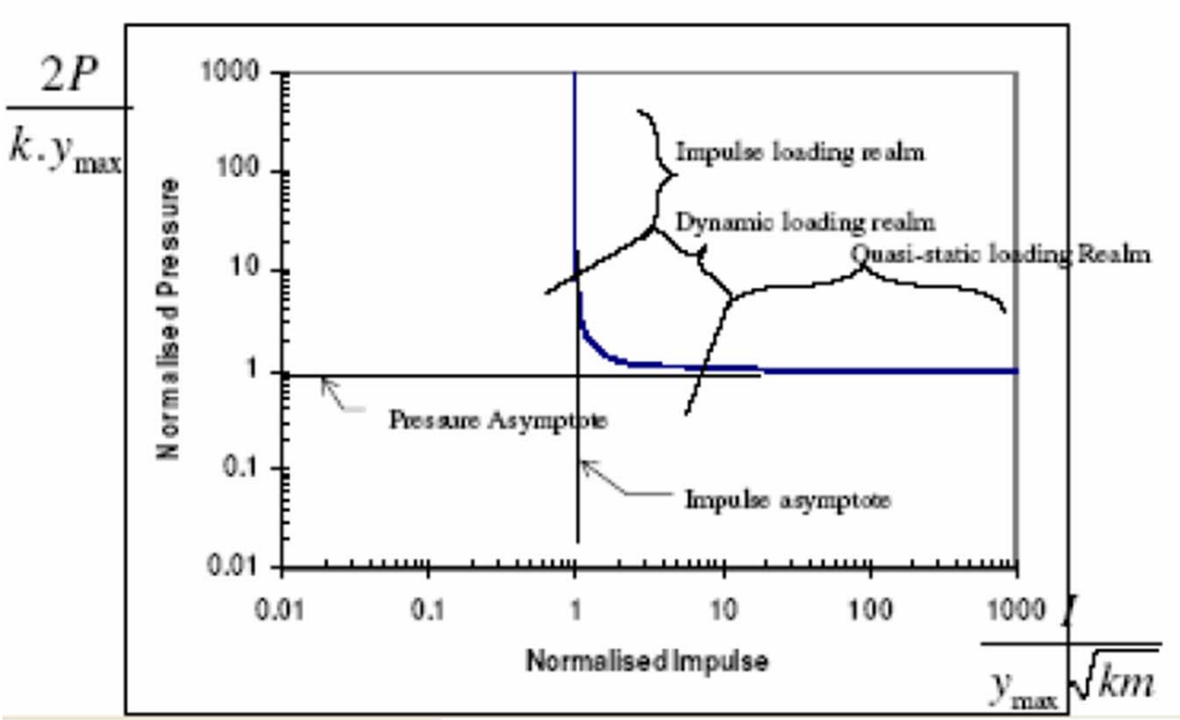
non-dimensional impulse

Hence impulse asymptote $\frac{u_m}{F/k} = \frac{I}{\sqrt{km}(F/k)}$ becomes:

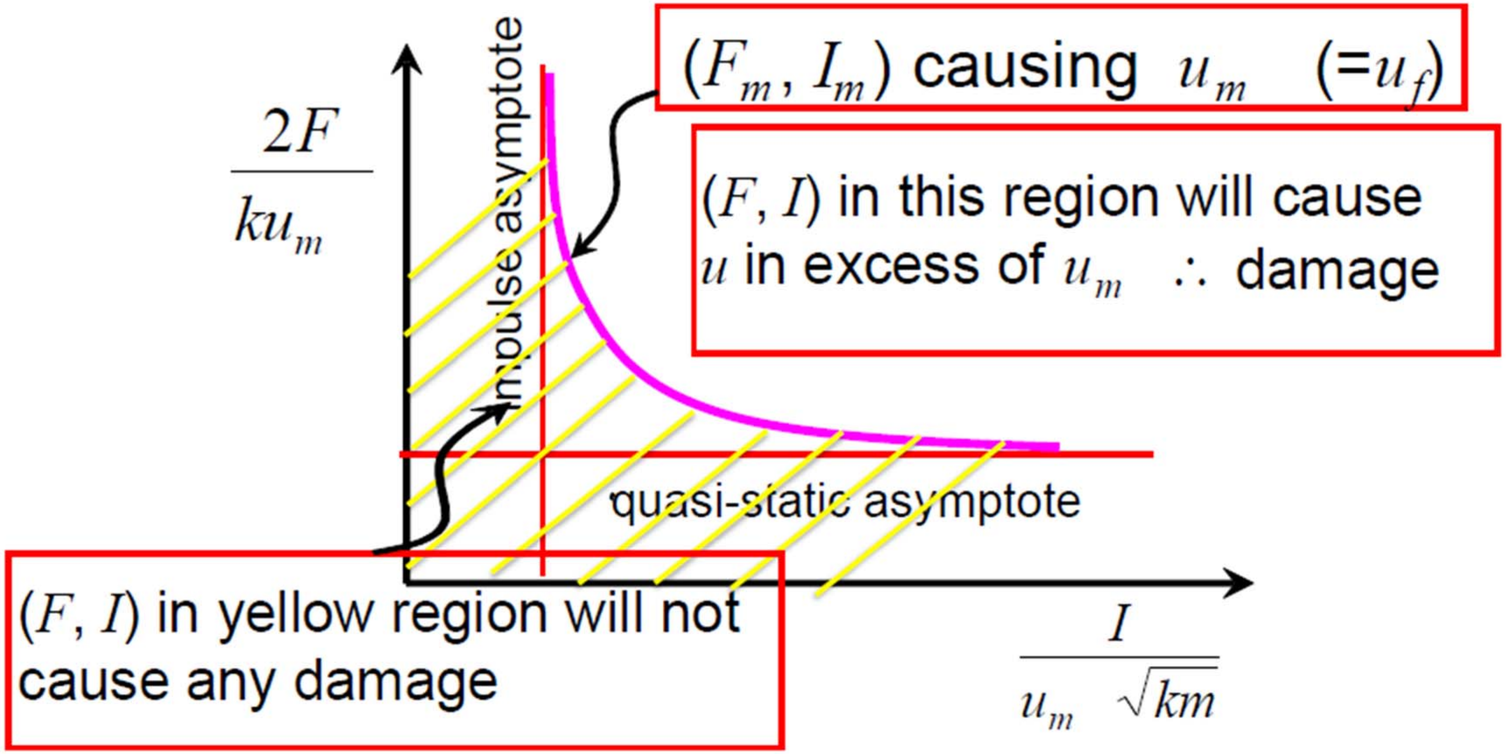
$$\frac{I}{u_m\sqrt{km}} = 1$$


10
1
0.1
0.01
0.001
umax/(F/k)
0.001
0.01
0.1
1
10
100
td / Tn
Quasi-static asymptote
Impulse asymptote
$\frac{2F}{ku_m}$
impulse asymptote
quasi-static asymptote
$\frac{I}{u_m \sqrt{km}}$

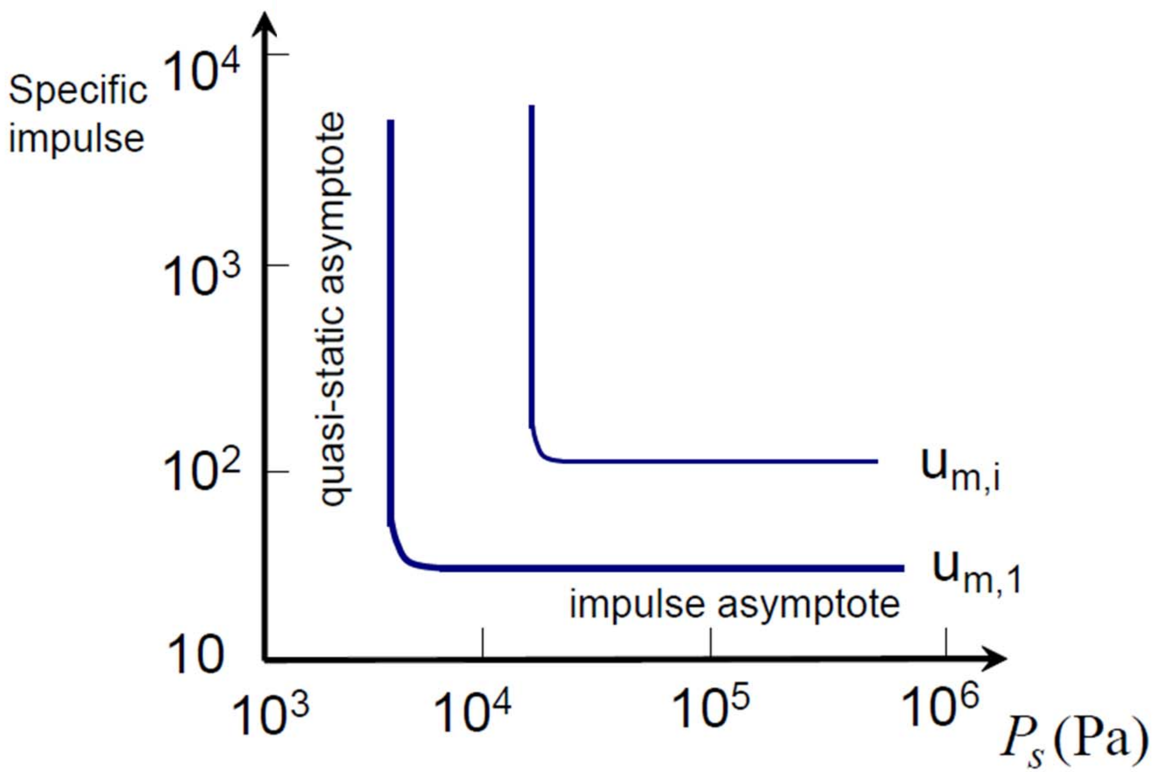
Pressure – impulse diagrams



Load Regime	Limits	
Impulsive	$\omega t_d < 0.4$	$t_d/T < 6.37 \times 10^{-2}$
Dynamic	$0.4 < \omega t_d < 40$	$6.37 \times 10^{-2} < t_d/T < 6.37$
Quasi-static	$\omega t_d > 40$	$t_d/T > 6.37$

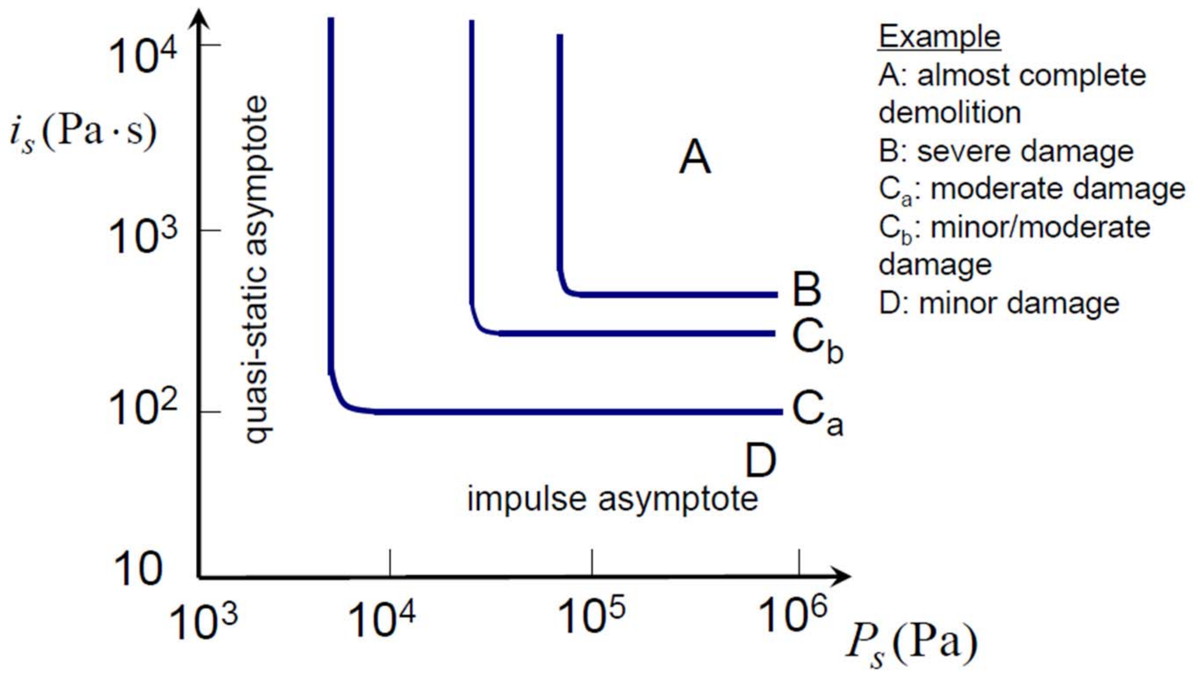

$\frac{2F}{ku_m}$
impulse asymptote
(F_m, I_m) causing u_m $(=u_f)$
(F, I) in this region will cause u in excess of u_m $\therefore$ damage
quasi-static asymptote
(F, I) in yellow region will not cause any damage
$\frac{I}{u_m \sqrt{km}}$

For a particular type of structure, diagram are presented in absolute impulse (specific) vs. overpressure terms, for different damage (u_m) levels



Example (for illustration only)

In practice, such diagrams are often constructed on empirical basis, not necessarily with explicit SDOF/limit displacement values



P-I diagram for damage to some small buildings

Typical Blast Damage to Structures

Pressure		Damage
(psi)	(kPa)	
0.02	0.14	Annoying Noise (137 dB), if of low frequency
0.03	0.21	Occasional Breakage of large glass windows already under strain
0.04	0.28	Loud Noise (143 dB). Sonic boom glass failure
0.10	0.70	Breakage of small windows under strain
0.15	1.0	Typical pressure for glass failure
0.30	2.1	Some damage to ceilings, limit of missiles
0.40	2.8	Limited minor structural damage
0.50-1.0	3.5-7.0	Large and small windows shattered, occasional damage to window frames
0.75	5.2	Minor damage to house structures; 20-50% roof tiles displaced
0.90	6.3	Roof damage to oil storage tanks
1.0	7.0	Partial demolition of houses, made uninhabitable

Source: Clancy, 1972, from WWII Data

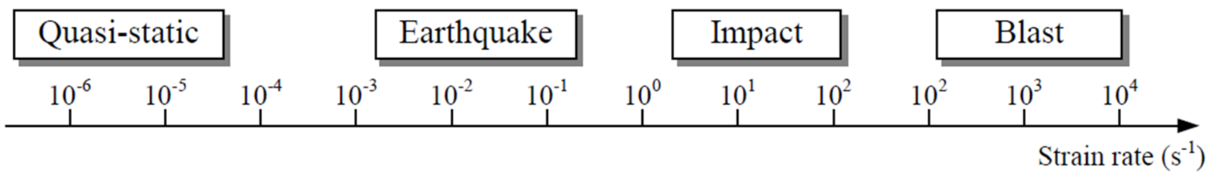
Typical Blast Damage to Structures

Pressure		Damage
(psi)	(kPa)	
5.0	35	Wooden utility poles damaged
7.0	49	Rail cars overturned
7-8.0	49-56	Brick panels (8-12"), not reinforced, fail by flexure
7-9	49-63	Collapse of steel girder framed building
7-10	49-70	Cars severely crushed
8-10	56-70	Brick walls completely demolished
9	63	Collapse of steel truss type bridges; loaded train wagons demolished
>10	>70	Complete destruction of all un-reinforced buildings
13	91	18" brick walls completely destroyed
70	490	Collapse of heavy masonry or concrete bridges

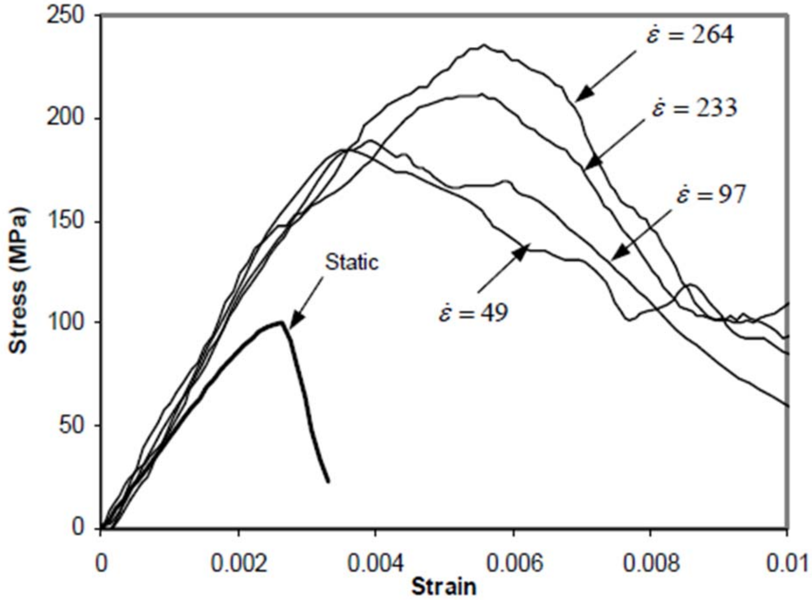
Source: Clancy, 1972, from WWII Data

Material behaviors at high strain rate

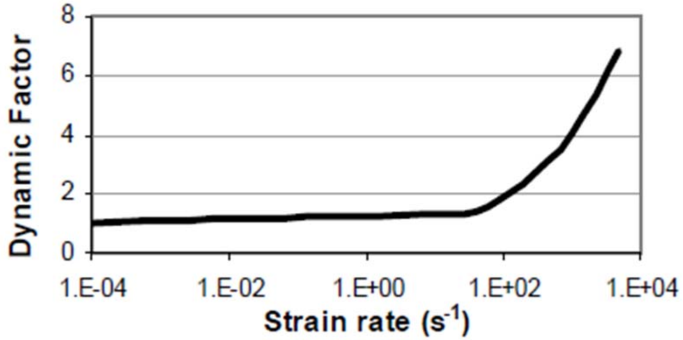
- Blast loads typically produce very high strain rates in the range of 10^2 - 10^4 s^{-1}.
- This high straining (loading) rate would alter the dynamic mechanical properties of target structures and, accordingly, the expected damage mechanisms for various structural elements.
- It can be seen that ordinary static strain rate is located in the range: 10^{-6}-10^{-5} s^{-1}, while blast pressures normally yield loads associated with strain rates in the range: 10^2-10^4 s^{-1}.
- For reinforced concrete structures subjected to blast effects the strength of concrete and steel reinforcing bars can increase significantly due to strain rate effects.
- The typical effects of increased strain rate on the response of structural steels are an increase in yield stress; an increase in ultimate strength, even smaller than for yield stress; and a reduction in the elongation at rupture



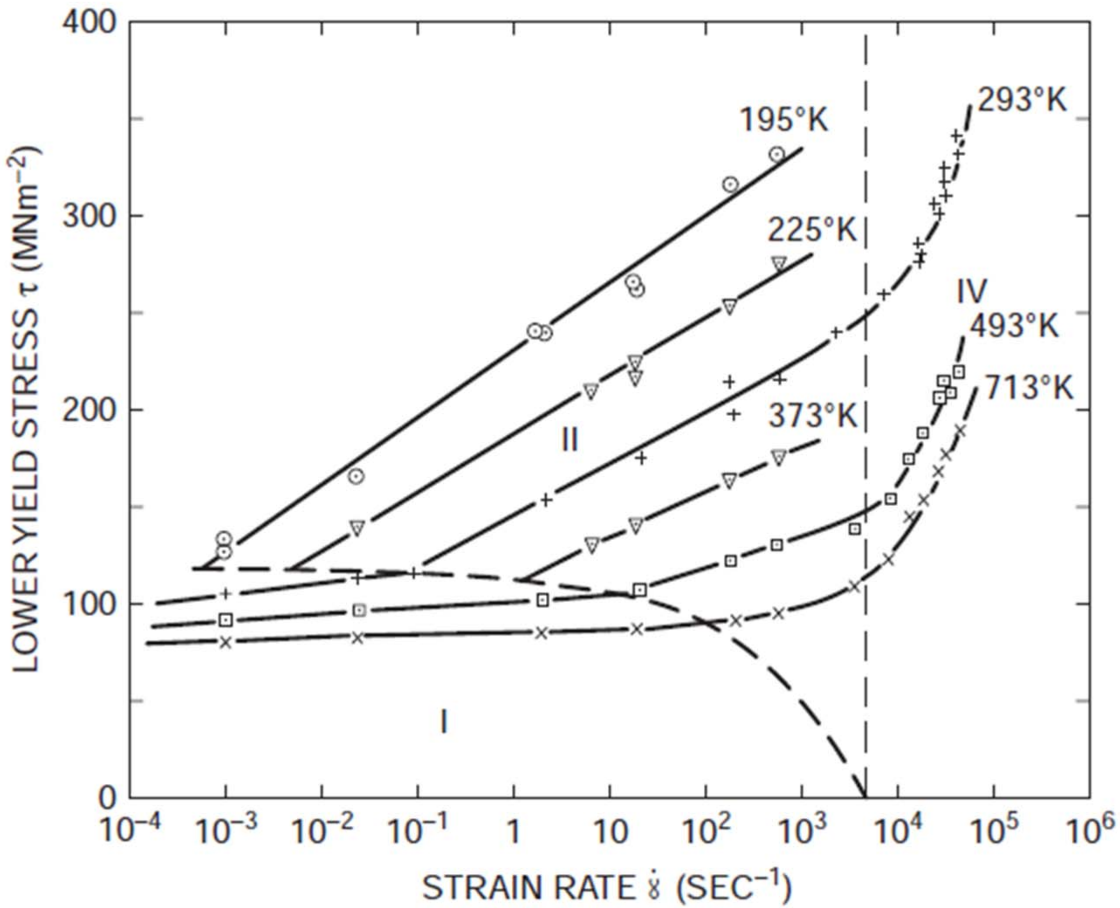
Strain rates associated with different types of loading



Stress-strain curves of concrete at different strain rates



Dynamic increase factor for peak stress of concrete



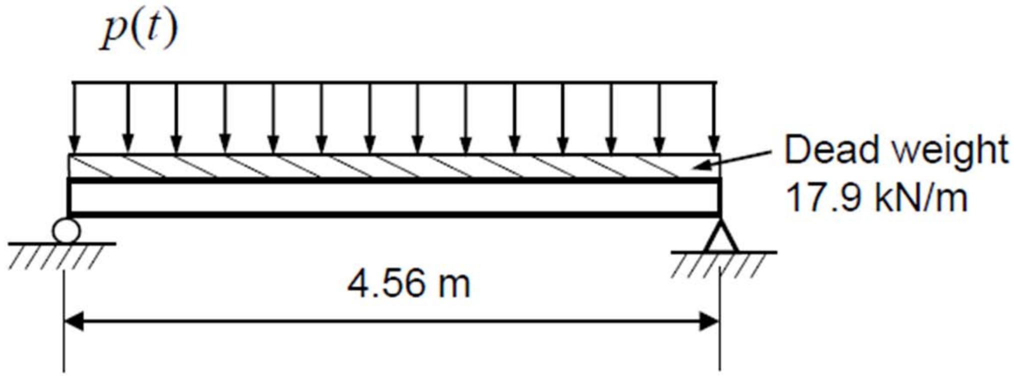
Effect of strain rate on mild steel

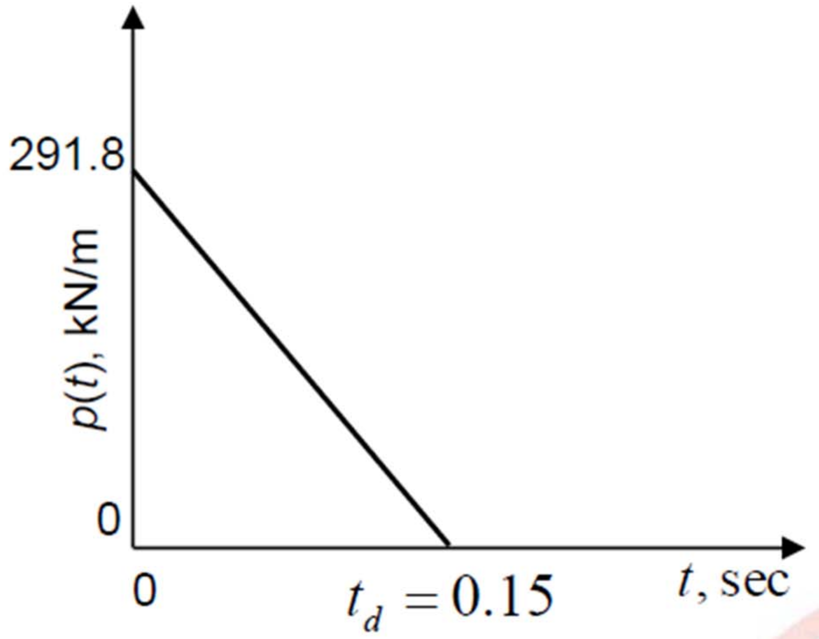
Design Examples for SDOF

- Ex 1 RC beam (elastic design and plastic design)
- Ex 2 Steel beam (elastic design and plastic design)

<u>Design Example 1</u>

- Design the beam for both elastic and elasto-plastic design response:
  - The following figures show an RC beam 4.56 m long, carrying a dead load of 17.9 kN/m and subjected to a triangular blast load of 291.8 kN/m with a duration of 0.15s.
  - Dynamic yield strength of steel f_{ykd}= 344.75 MPa
  - Dynamic concrete compressive strength f_{ckd} = 27.6MPa
  - Steel ratio of RC beam ρ_s = 0.015
  - Assume dynamic strength is 25% greater than static strength


p(t)
Dead weight
17.9 kN/m
4.56 m


291.8
p(t), kN/m
0
0
t_d = 0.15
t, sec

EN 1992-1-1:2004 (E)

3.1.7 Stress-strain relations for the design of cross-sections

(3) A rectangular stress distribution (as given in Figure 3.5) may be assumed.

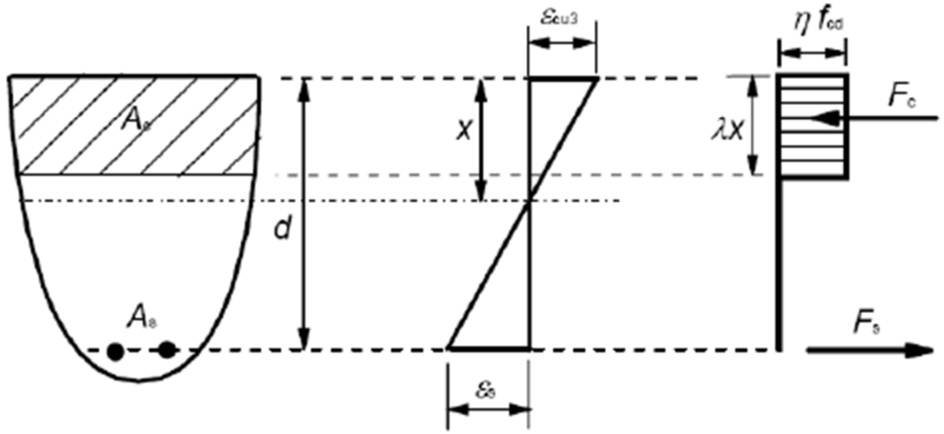


Figure 3.5: Rectangular stress distribution

$\lambda = 0,8$ for $f_{ck} \leq 50$ MPa
$\lambda = 0,8 - (f_{ck} - 50)/400$ for $50 < f_{ck} \leq 90$ MPa

and

$\eta = 1,0$ for $f_{ck} \leq 50$ MPa
$\eta = 1,0 - (f_{ck} - 50)/200$ for $50 < f_{ck} \leq 90$ MPa

3.1.6 Design compressive and tensile strengths

(1)P The value of the design compressive strength is defined as

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C \quad (3.15)$$

where:

γ_C is the partial safety factor for concrete, see 2.4.2.4, and

α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied.

(α_{cc}**=0.85** in UK and Singapore national Annex)

2.4.2.4 Partial factors for materials

(1) Partial factors for materials for ultimate limit states, γ_C and γ_S should be used.

Note: The values of γ_C and γ_S for use in a Country may be found in its National Annex. The recommended values for 'persistent & transient' and 'accidental, design situations are given in Table 2.1N. These are not valid for fire design for which reference should be made to EN 1992-1-2.

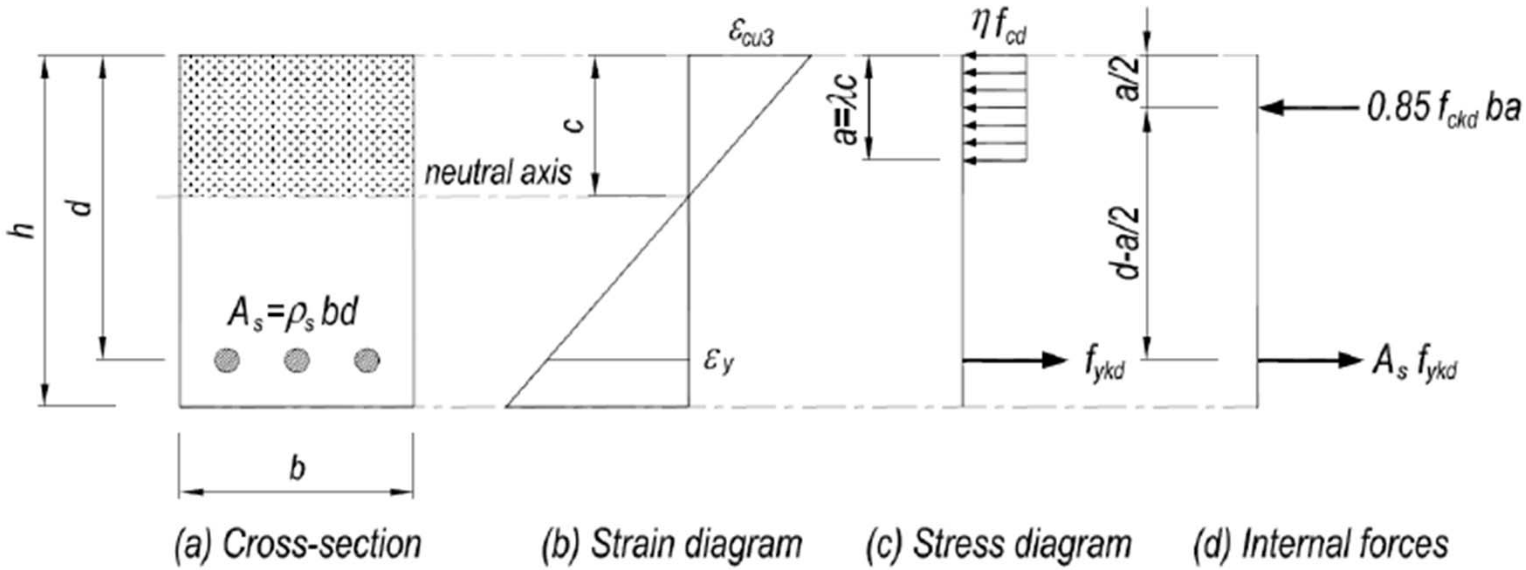
For fatigue verification the partial factors for persistent design situations given in Table 2.1N are recommended for the values of $\gamma_{C,fat}$ and $\gamma_{S,fat}$.

Table 2.1N: Partial factors for materials for ultimate limit states

Design situations	γ_C for concrete	γ_S for reinforcing steel	γ_S for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

Dynamic compressive strength: $f_{ckd}=f_{ck}/1.2$

Cross-sectional analysis of the investigated beam



For $f_{ck} \leq 50$ MPa → $\lambda=0.8;\ \eta=1.0$ and $f_{cd}=\alpha_{cc} f_{ckd}=0.85 f_{ckd}$

- From equilibrium of compression and tension forces across a section, we can obtain neutral axis depth "*a*":

$$0.85 f_{ckd} b a = A_s f_{ykd} \Rightarrow a = \frac{A_s f_{ykd}}{0.85 f_{ckd} b}$$

- Taking moment about centroid of compression force,

$$M_p = A_s f_{ykd} d\left(1 - \frac{a}{2d}\right) = A_s \sigma_y d\left(1 - \frac{A_s f_{ykd}}{1.7 f_{ckd} b d}\right)$$

- Knowing $\rho_s = A_s/(bd)$,

$$M_p = \rho_s b d^2 f_{ykd}\left(1 - \frac{\rho_s f_{ykd}}{1.7 f_{ckd}}\right)$$

Elastic design

Flexure formula for RC section

$$M_p = \rho_s b d^2 f_{ykd}\left(1 - \frac{\rho_s f_{ykd}}{1.7 f_{ckd}}\right)$$

Required moment of resistance in terms of width and depth of section bd^2,

$$M_p = 0.015 \times bd^2 \times 344.75 \times 10^3 \times \left(1 - \frac{0.015 \times 344.75 \times 10^3}{1.7 \times 27.6 \times 10^3}\right) = 5.114 \times 10^3 \times bd^2 (\mathrm{kNm})$$

From Table 5.1 for beams subjected to u.d.l. and within elastic strain range, maximum resistance is:

$$R_m = \frac{8M_P}{L}$$

∴ Max resistance is $R_m = \frac{8M_p}{L} = \frac{8 \times 5.114 \times 10^3 \times bd^2}{4.56} = 8962 \times bd^2 \,(\text{kN})$

Assuming T=0.05 s, $t_d/T \approx 3$.

From Fig. 2.7(a) for elastic system with triangular load, DLF $\approx$1.8

Assuming a self-weight of 11.7 kN/m in addition to 17.5 kN/m from slab attached to the beam,

$$\text{Required } R_m = 1.8 \times 291.8 \times 4.56 + (11.7 + 17.5) \times 4.56 = 2528.2\,\text{kN}$$

$$\text{Required } bd^2 = \frac{2528.2}{8962} = 0.28\,\text{m}^3$$

Trial beam size:

***b* = 460 mm** and ***d* = 830 mm**

Average ***I*** of cracked and uncracked concrete:

$$I_a = \frac{bd^3}{2}(5.5\rho_s + 0.083)$$

$$I_a = \frac{0.46 \times 0.83^3}{2} \times (5.5 \times 0.015 + 0.083) = 0.022(\mathrm{m}^4)$$

Average Elastic flexural stiffness of RC beam (Table 5.1):

$$k = \frac{384EI}{5L^3} = \frac{384 \times (2.1 \times 10^6) \times 0.022}{5 \times 4.56^3} = 0.371 \times 10^6 \text{ kN/m}$$

To allow for concrete cover to rebar say d =0.95h, total weight of beam selected is 24x0.46x(0.83/0.95)=9.6 kN/m

Dead Weight of RC beam + slab = (9.6+17.5)x4.56=123.6 kN

M_t= 123.6/9.81 = 12.6 kN.sec^2/m

From Table 5.1, load-mass factor of equivalent SDOF K_{LM}=0.78

$$T = 2\pi\sqrt{\frac{K_{LM}M_t}{k}} = 2\pi\sqrt{\frac{0.78\times12.6}{0.371\times10^6}} = 0.032(\text{sec})$$

Based on a more accurate estimate of *T* and from Fig. 2.7(a),

$$\frac{t_d}{T} = \frac{0.15}{0.032} = 4.5$$

$$(\mathrm{DLF})_{\max} = 1.89$$

$$\text{Required } R_m = 1.89 \times 291.8 \times 4.56 + 124.12 = 2639(\mathrm{kN})$$

Actual R_m = 2528 kN < required R_m of 2639 kN

You may iterate one more round by increasing *b* and *d* slightly!

For this example, just adopt R_m = 2528 kN

To obtain dynamic reaction forces, refer to Table 5.1.

Need to find t_m the time of max response:

Fig. 2.7(b), with $t_d/T = 4.5$, $t_m/T = 0.487$ or $t_m = 0.016$ s.

Fig. 5.12, dynamic load $p(t)$ per unit length = 261 kN/m

$$V = 0.39 R_m(\text{live}) + 0.11F + \text{dead}$$
$$= 0.39 \times 2528 + 0.11 \times 261 \times 4.56 + \frac{1}{2} \times 123.6$$
$$= 1{,}178.6\text{kN}$$

Web reinforcement should be designed to withstand this dynamic shear force!

Elasto-plastic design

For elasto-plastic design of RC beam, set μ = 3.

Since load duration is relatively long, we can use Q-S limit obtained from:

work done by external force = strain energy

$$\text{Required } R_m = F_1\left(\frac{1}{1-1/2\mu}\right) = 291.8\times 4.56\times 1.2 = 1597\text{kN}$$

Assume a lower value of R_m & est. beam weight of 4.38 kN/m

$$\text{Required } R_m = 291.8\times 4.56 + (17.5+4.38)\times 4.56 = 1430\text{kN}$$

$$\text{Required } bd^2 = \frac{1430}{8000} = 0.179\text{m}^3$$

Choose *b* = 0.38 m and *d* = 0.68 m

$$\text{Beam weight} = 24 \times 0.38 \times \frac{0.68}{0.9} = 6.89\,\text{kN/m}$$

$$\text{Totalweight} = (6.89 + 17.5) \times 4.56 = 111.2\,\text{kN}$$

$$M_t = 111.2 / 9.81 = 11.34\,\text{kN.sec}^2/\text{m}$$

$$I_a = \frac{0.38 \times 0.68^3}{2}(5.5 \times 0.015 + 0.083) = 0.01\,\text{m}^4$$

$$k = \frac{384 E I_a}{5L^3} = \frac{384 \times (2.1 \times 10^6) \times 0.0068}{5 \times 4.56^3} = 16.8 \times 10^6\ \text{kN/m}$$

From Table 5.1 for elastic and plastic strain range,

K_{LM} = 0.78 and 0.66. Take the average ≈ 0.70

$$T = 2\pi\sqrt{\frac{K_{LM}M_t}{k}} = 2\pi\sqrt{\frac{0.70\times 11.34}{16.8\times 10^6}} = 0.043\text{sec}$$

$$\frac{t_d}{T} = \frac{0.15}{0.043} = 3.5$$

$$\frac{R_m(\text{net})}{F_1} = \frac{1430}{291.8\times 4.56} = 1.07$$

$$V_{\max} = 0.39R_m(\text{net}) + 0.11F + \text{dead}$$

$$= 0.39\times 1430 + 0.11\times 291.8\times 4.56 + \frac{1}{2}\times(17.5+4.38)\times 4.56$$

$$= 754\text{kN}$$

Comparison of Design Example 1

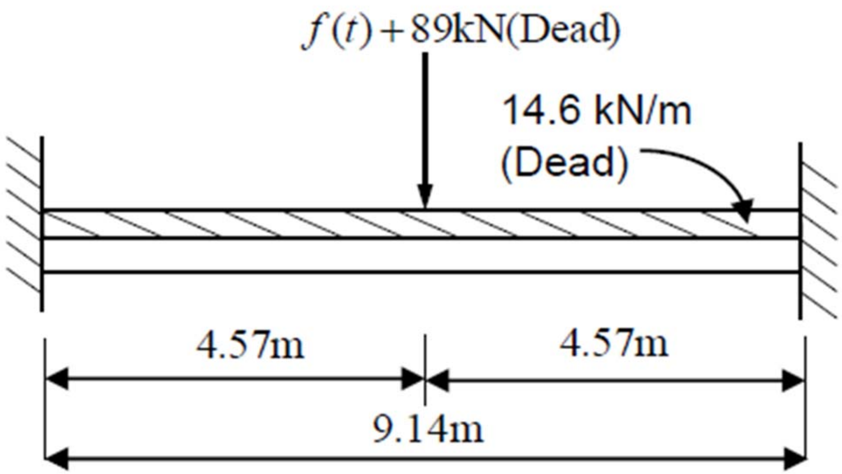
	Elastic design	Plastic design
RC Beam dimensions	b = 460 mm d = 830 mm	b = 380 mm d = 680 mm
Maximum resistance R_m	2528 kN	1430 kN
Dynamic reaction V_{max}	1178 kN	754 kN

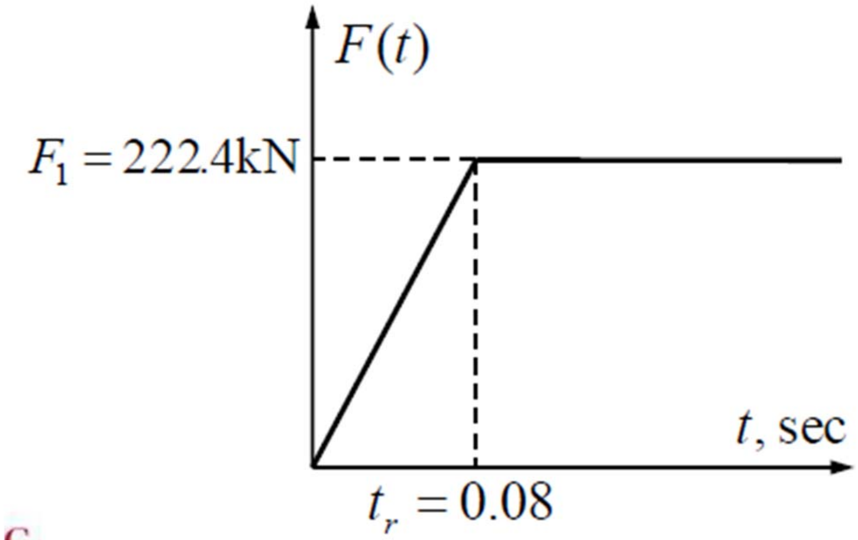
Design Example 2

Design a steel beam for elastic design response:

A steel beam of 9.14 m length is clamped at both ends. For elastic design, the bending stress should be less than 207 MPa.

The beam is subjected to dead loads of 14.6 kN/m and 89 kN at mid span.
The beam is braced at load point so that there is no LTB. It is subjected to a dynamic point load F(t) at mid-span as shown in the figures.


$f(t)+89$kN(Dead)
14.6 kN/m
(Dead)
4.57m
4.57m
9.14m


$F(t)$
$F_1 = 222.4$kN
t, sec
$t_r = 0.08$

Elastic Design

Assume $t_r/T \approx 2/3$; from Fig. 2.9, DLF $\approx$ 1.4. This only affects the dynamic point load $P(t)$ but not the dead point

$$M_{\max} = \frac{w_D L^2}{12} + \frac{F_D L}{8} + \frac{F_1 L}{8}(\mathrm{DFL})_{\max}$$

$$M_{\max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.4 = 559.1\mathrm{kNm}$$

Required Elastic Section Modulus:

$$W_{el-req} = \frac{M_{\max}}{\sigma} = \frac{559.1}{207 \times 10^3} = 2.7 \times 10^{-3}\,\mathrm{m}^3$$

Most economical section is 24WF76 (UB 610x229x113)

having I=8.72x10^{-4} m^4 and W_{el}=2.87x10^{-3} m^3

$$k = \frac{192EI}{L^3} = \frac{192 \times (210 \times 10^6) \times 8.72 \times 10^{-4}}{9.14^3} = 46{,}047\mathrm{kN/m}$$

From Table 5.2, for elastic clamped beams subjected to mid-span point load,

$K_L = 1.0 \quad K_M = 1.0$ *concentrated mass (for point load)*

$K_M = 0.37$ *distributed mass (for dead load u.d.l.)*

$$M_e = \sum K_M M = \frac{89 \times 1.0 + 14.6 \times 9.14 \times 0.37}{9.81} = 14.11\,\text{kNsec}^2/\text{m}.$$

$$K_e = K_L k = 46{,}047 \times 1.0 = 46{,}047\,\text{kN/m}$$

$$T = 2\pi\sqrt{\frac{M_e}{K_e}} = 2\pi\sqrt{\frac{14.11}{46{,}047}} = 0.111\,\text{sec}$$

$$\frac{t_d}{T} = 0.08/0.111 = 0.72$$

2nd round iteration from Fig. 2.9,

$(DLF)_{max}$=1.35

$$M_{max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.35 = 546.3\text{kNm}$$

Required Elastic Section Modulus:

$$\sigma_{max} = \frac{M_{max}}{W_{el}} = \frac{546.3}{2.87 \times 10^{-3}} = 190.3 \times 10^3 \text{kN/m}^2 = 190.3\text{MPa}$$

From Fig. 2.9(a), $t_r/T=0.72$, DLF = 1.35

$\Rightarrow R_m$ =1.35 x 222.4 = 300.2 kN

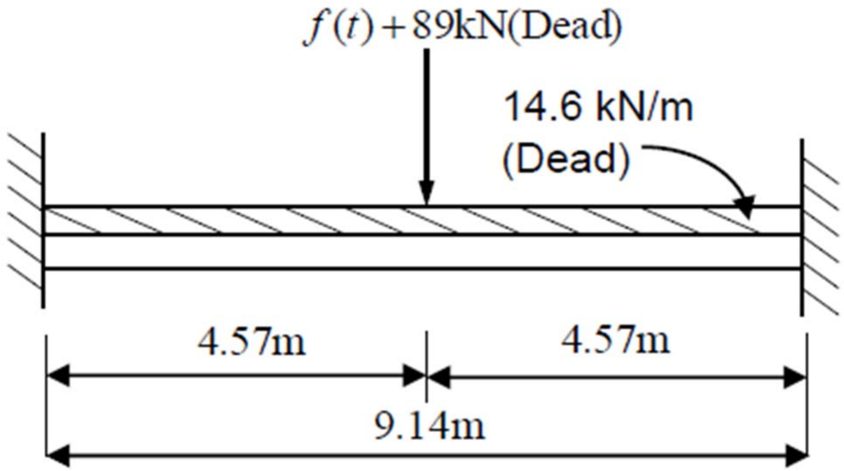
$$V_{max} = 0.71 \times 300.2 - 0.21 \times 222.4 + \text{dead}$$
$$= 213.1 - 46.7 + 89 \times 0.5 + 14.6 \times 9.14 \times 0.5 = 277.6\,\text{kN}$$

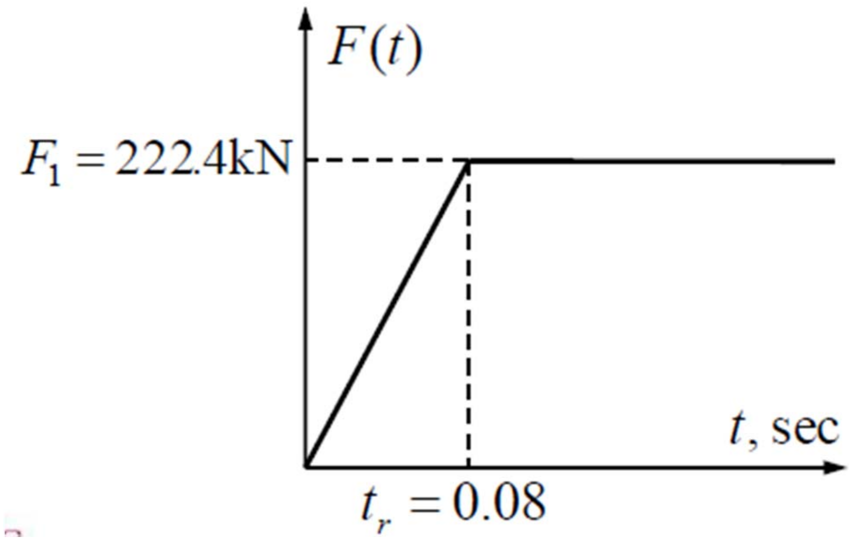
Elasto-plastic design

Design a steel beam for plastic design response:

A steel beam of 9.14 m length is clamped at both ends. For plastic design, the bending stress should be less than its yield strength of 344 MPa

The beam is subjected to dead loads of 14.6 kN/m and 89 kN at mid span.
The beam is braced at load point so that there is no LTB. It is subjected to a dynamic point load F(t) at mid-span as shown in the figures.


$f(t) + 89\text{kN(Dead)}$
14.6 kN/m
(Dead)
4.57m
4.57m
9.14m


$F(t)$
$F_1 = 222.4\text{kN}$
t, sec
$t_r = 0.08$

Plastic Design

Assume $t_r/T \approx 2/3$ and from Fig. 2.9, DLF $\approx$ 1.4. This only affects the dynamic point load $P(t)$ but not the dead point

$$M_{\max} = \frac{w_D L^2}{12} + \frac{F_D L}{8} + \frac{F_1 L}{8}(\mathrm{DFL})_{\max}$$

$$M_{\max} = \frac{14.6 \times 9.14^2}{12} + \frac{89 \times 9.14}{8} + \frac{222.4 \times 9.14}{8} \times 1.4 = 559.1\,\mathrm{kNm}$$

Required Plastic Section Modulus:

$$W_{pl-req} = \frac{M_{\max}}{\sigma} = \frac{559.1}{344 \times 10^3} = 1.63 \times 10^{-3}\,\mathrm{m}^3$$

Most economical section is UB 533x210x82 having I=4.75x10^{-4} m^4 and W_{pl}=2.06x10^{-3} m^3 and Moment capacity:

$$M_s = M_m = W_{pl} \times \sigma = 2.06 \times 344 = 708.6\,\mathrm{kNm}$$

From Table 5.2, for plastic clamped beams subjected to mid-span point load, maximum resistance is:

$$R_m = 4(M_s + M_m)/L = 4 \times 2 \times 708.6/9.14 = 620\,\text{kN}$$

Dynamic reaction is:

$$V_{\max} = 0.75 \times 620 - 0.25 \times 222.4 + \text{dead}$$
$$= 465 - 55.6 + 0.5 \times 89 + 0.5 \times 14.6 \times 9.14 = 520.6\,\text{kN}$$

Comparison of Design Example 2

	Elastic design	Plastic design
Beam chosen	UB 610x229x113	UB 533x210x82
Maximum resistance	300.2 kN	620 kN
Dynamic reaction	277.6 kN	520.6 kN

This lecture was prepared for the 1st Edition of SUSCOS (2012/14) by Kang-Hai TAN, Nanyang Technological University, Singapore and ULg

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